

CONSTITUTIVE LAWS FOR COMPRESSION CONCRETE UNDER MONOTONIC AND CYCLIC LOADING: CHARACTERISTIC MODELS

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ABSTRACT

This paper reviews the existing characteristic models for stress-strain of concretes subjected to compression monotonic and cyclic loading. The existing compressive models depend on the discoveries of the performed tests and modeling methods and mainly are found to have a good accuracy when compared to the experimental test results. Some of the models are based on the experimental investigation while some are based on analytical investigations. Some models are modified by other researchers to include some aspects that were neglected. Stress-strain models are defined by considering some components such as envelope curve, unloading and reloading curves. Some of the models are proposed for the confined or unconfined concretes or even both.

Keywords; concrete, monotonic loading, cyclic loading, envelope curve, unloading, reloading.

INTRODUCTION

Concrete is a composite material having a mixture of water, cement, fine and coarse aggregates sometimes with the addition of admixtures to tweak the certain properties. The performance of concrete relies upon the nature of the constituent materials. The knowledge of stress-strain behavior and fracture mechanism of concrete constituents is needed to forecast how concrete behaves under various load types (Washa, 1950). Concrete is used in the various structural application and such structures are subjected to various compressive and tensile forces, this can be in a form of monotonic, cyclic or sustained loading. Creep occurs as a result of sustained loading; a slow but continuous process leading to deformation of concrete (Youssef and Moftah, 2007). Studying the stress-strain diagram of reinforced concrete (RC) elements subjected to compression make it possible to determine the properties of RC members such as ductility and strength (Chen, 2013). This knowledge can only be obtained through thorough testing and monitoring so that the performance can be studied and new theories can be introduced (Washa, 1950). However, in the research by Claisse and Dean (2013); and Martin and Terry (1991) it shows that a sustained load producing low stresses applied prior to testing monotonically, and significantly increases the initial modulus of elasticity of concrete. This implies that creep, at least at low stresses, is not a result of detrimental cracking or damage to the material. It shows that a maintained load creating low stresses connected prior to testing monotonically, and fundamentally expands the underlying modulus of elasticity of concrete. This infers creep at low stresses, is not an aftereffect of adverse splitting or harm to the material.

Recently, the introduction of many understandable and more complex models under numerous stress states to explain the behavior of concrete has been on the increase. However, most of these developed models have larger theoretical significance than practical importance

but they solely explain some selected parts of concrete behavior and their application is limited to the basic pragmatic application. Vital highlights of the constitutive model of a concrete ought to incorporate not just the basically precise description of the real behavior of concrete yet, in addition, the clearness of formulation and proficient application in a vigorous and stable nonlinear state determination algorithm (Aslani and Jowkarmeimandi, 2012). (Sadeghi, 1994, 1995, 2001) produced a finite element model to eliminate the issue attributed to scale impact named "Biaxial Bending Column Simulation" where the column models were assessed based on the practical and simulated test. Along these lines, the presented non-linear law for stress-strain regarding confined concrete subjected to load of cyclic nature was changed and approved.

In a research conducted by Konstantinidis et al. (2004), an analytical model was developed for when a cyclic load is applied to a concrete and it is found out that what is more significant is the precise description of envelope curve instead of the pattern of reloading and unloading branches. Verification of this can be seen from the stress-strain graph of a square sample having a strength of 57 MPa under frequent uniaxial compression, along with the stress-strain diagram of the same square sample subjected to increasing monotonic compressive load. In addition, it can be observed that the curve of the stress-strain diagram of a high strength concrete subjected to cyclic loading meets the stress-strain curve of a monotonic loading. A study by Karsan and Jirsa (1969) reported a similar outcome for concrete of normal strength. Other criterions that need to be considered are the increase in the plastic strain of concrete along with the impact of confinement because of transverse reinforcement.

A basic model was introduced for normal strength concrete in tension and compression under cyclic loading. Specific importance towards the definition of strength and stiffness debasement was delivered by the cyclic load in compression and tension, how reloading and unloading curves behave and how crack's initiation and closing takes place (Sima, Roca, and Molins, 2008). However, because of increasing load two separate parameters of damage in tension and compression to model concrete deterioration were developed by Aslani and Jowkarmeimandi (2012).

The presented models can be applied in combination with finite element analysis to simulate many types of RC elements subjected to cyclic and monotonic loading condition (Sadeghi, 2014).

Existing Characteristic Models for Monotonic and Cyclic Loading

Descriptions of the findings of the researchers are reviewed. There are some elements of concrete stress-strain model under monotonic and cyclic loading namely; envelope curve, unloading and reloading curve.

Models of Sadeghi and Nouban

The various developed simple stress-strain models for confined concrete under compression by Sadeghi (1995, 2002 and 2014) and Sadeghi and Nouban (2010a, 2017a) are suitable in the simulation of beams, columns, and beam-columns subjected to monotonic and cyclic loading. This model is based on previous models but modified after being validated by some experimental tests carried by researchers on RC columns under axial load and mono-axial and biaxial bending moment. (Kabir Sadeghi and Nouban, 2010) found out that the developed model can be used together with finite element analysis to simulate a vast array of RC elements subjected to different kinds of loading condition.

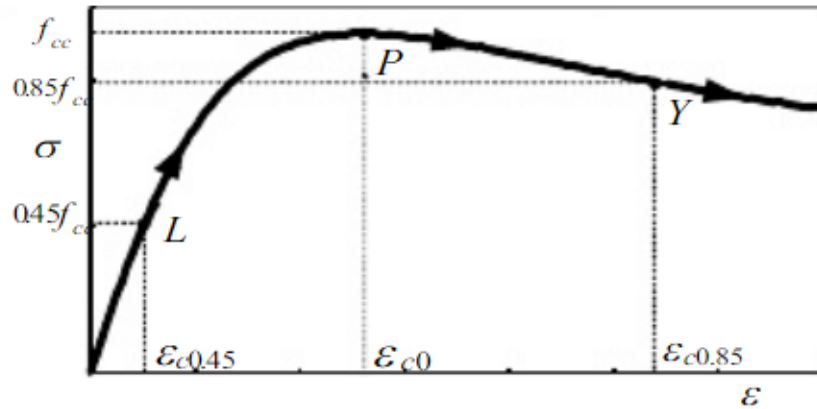


Figure 1. Stress-strain diagram of confined concrete subjected to monotonic loading (Kabir Sadeghi and Nouban, 2010)

Eqs. (1-3) are proposed by Sadeghi and Nouban to find the stress in the elements under monotonic loading, unloading and reloading.

Envelope Curve:

$$\sigma = \frac{f_{cc}}{A_L \left(\frac{\epsilon}{\epsilon_{c0}}\right)^2 + B_L \left(\frac{\epsilon}{\epsilon_{c0}}\right) + C_L + D_L \left(\frac{\epsilon}{\epsilon_{c0}}\right)^{-1}} \quad (1)$$

To find unknowns A_L , B_L , C_L and D_L , values of stress and strain at L, P and Y points. At the peak of the curve is slope is equal to zero.

Unloading Curve

$$\sigma = \left[A_u \left(\frac{\epsilon}{\epsilon_{c0}}\right)^3 + B_u \left(\frac{\epsilon}{\epsilon_{c0}}\right)^2 + C_u \left(\frac{\epsilon}{\epsilon_{c0}}\right) + D_u \right] f_{cc} \quad (2)$$

The unknowns of A_U , B_U , C_U , and D_U is found from the unloading curve, it is determined by inserting the coordinates at the beginning point of unloading and the highest point on the curve with zero stress.

Reloading Curve

$$\sigma = \left[A_R \left(\frac{\epsilon}{\epsilon_{c0}}\right)^3 + B_R \left(\frac{\epsilon}{\epsilon_{c0}}\right)^2 + C_R \left(\frac{\epsilon}{\epsilon_{c0}}\right) + D_R \right] f_{cc} \quad (3)$$

The unknowns A_R , B_R , C_R , and D_R is found from the reloading curve, it is determined by inserting the coordinates at the beginning point of reloading and the highest point on the envelope curve.

For the definition of the symbols, please refer to the notation Section at the end of this paper.

The details of these formulas are given in the references: ((Sadeghi, 1995, 2002, 2014); (Sadeghi and Nouban, 2010a, 2017a) and the application of these formulas are given in the references: ((Sadeghi, 1998, 2011a, 2011b, 2015, 2017a, 2017b); (Sadeghi and Nouban, 2010b, 2017b, 2017c, 2018); (Sadeghi and Sadeghi, 2013) and (Hashemi et al., 2018).

Models of Karsan and Jirsa

A research was carried by Karsan and Jirsa (1969) to observe how concrete behave when concrete subjected to various compressive loadings, they made use of rectangular concrete columns as a specimen and were subjected to various loading regimes, and these are;

- Monotonic progressing load until rupture

- Cycles to the envelope curve
- Cycles to envelope curve, each cycle affixing a certain strain addition
- Cycles amid minimum and maximum levels of stress

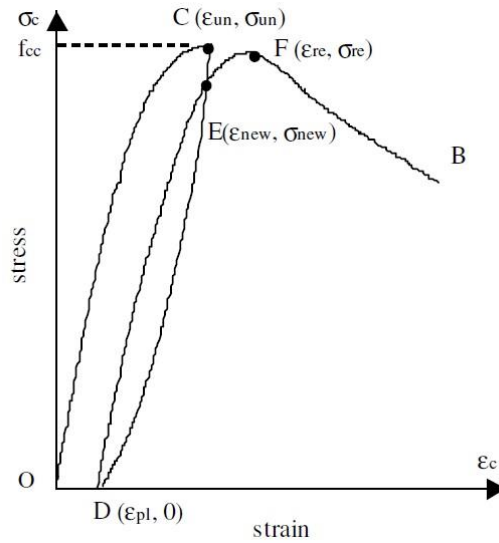


Figure 2. Stress-strain diagram of cyclically loaded concrete (Karsan and Jirsa, 1969)

After some practical tests on short columns which are rectangular in shape, Karsan and Jirsa (1969) discovered a locus of collective points in the stress-strain curve characterized by the crossing of the branches of loading and reloading. Most models accepted the mentioned discovery except model developed by Blakeley and Park (1973). As shown in the model confinement issue is not regarded, the unloading branch follows through the strain reversal point C i.e. unloading starting point, point E and the point of complete reversal D. Similarly, the reloading branch follows through the point of complete reversal, point E and the point of return F. They conclude that unloading and reloading curves are distinct but are based on past load history. The strain equivalent to zero stress on the curves of reloading/unloading is regarded as plastic strain or non-recoverable strain, and significantly affects the structure of the curves of reloading and unloading (Aslani and Jowkarmeimandi, 2012).

Envelope Curve

$$\sigma = 0.85f_c \frac{\varepsilon}{\varepsilon_{ccl}} e^{\left(1 - \frac{\varepsilon}{\varepsilon_{ccl}}\right)} \tag{4}$$

Unloading Curve

$$\sigma = f_c \left[0.093 \left(\frac{\varepsilon_{un}}{\varepsilon_{ccl}} \right)^2 + 0.091 \left(\frac{\varepsilon_{un}}{\varepsilon_{ccl}} \right) \right] \tag{5}$$

Reloading Curve

$$\sigma = f_c \left[0.145 \left(\frac{\varepsilon_{un}}{\varepsilon_{ccl}} \right)^2 + 0.13 \left(\frac{\varepsilon_{un}}{\varepsilon_{ccl}} \right) \right] \tag{6}$$

For the definition of the symbols, please refer to the notation Section at the end of this paper. The details of these formulas are given in the references: ((Karsan and Jirsa, 1969); (Aslani and Jowkarmeimandi 2012)) and the application of these formulas are given in the references: ((Zhang et al., 2018); (Konstantinidis et al., 2004); (Youssef and Moftah, 2007)).

The details of these formulas are given in the references: ((Blakeley and Park, 1973); (Aslani and Jowkarmeimandi 2012)) and the application of these formulas are given in the references: (Konstantinidis et al., 2004)).

Models of Martinez-Rueda

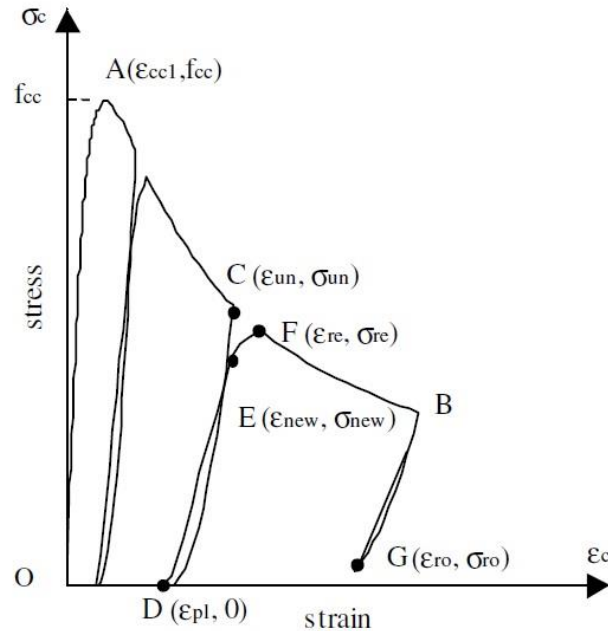


Figure 4. Stress-strain diagram of concrete under cyclic loading (Martinez-Rueda, 1997)

Martinez-Rueda (1997) did some modifications to Mander's et al. (1989) model on the fact that the experienced absence of numerical balance of Mander's model for huge displacements, that results to convergence issues if which caused joining issues when executed into a non-straight program following a fiber component approach. In addition, the plastic strain of low, halfway and the high strain was modified and taking into account the relieving of concrete as strain advances. The strains greater than ϵ_{un} was converted to a line that falls in between the strong point and point of return. To make up for the absence of a steady change between the branches of reloading and the envelope, the average value was given to returning strain between ϵ_{new} and ϵ'_{re} , which is gotten utilizing the experimental equation of (Karsan and Jirsa, 1969).

Envelope Curve

$$\sigma = \frac{f'_{cc} x^r}{r-1+x^r} \tag{9}$$

Where:

$$x = \frac{\epsilon_c}{\epsilon_{ccl}} \tag{10}$$

$$r = \frac{E_c}{E_c - E_{cl}} \tag{11}$$

Reloading curve

$$\sigma_{new} = \frac{0.9 f_{cc} \frac{\epsilon_c}{0.9 \epsilon_{cc}} - r}{r-1+x^r} \tag{12}$$

Unloading Curve

$$\sigma = \sigma_{un} \left(\frac{\epsilon - \epsilon_{pl}}{\epsilon_{un} - \epsilon_{pl}} \right)^2 \tag{13}$$

Reloading Curve

$$\text{First branch: } \sigma = \sigma_{ro} + E_r(\varepsilon - \varepsilon_{ro}) \tag{14}$$

$$\text{Second branch: } \sigma = \sigma_{new} + E_{re}(\varepsilon - \varepsilon_{re}) \tag{15}$$

$$\sigma_{cr} = \frac{2.5f_{cc}r}{r-1+2.5r} \tag{16}$$

For the definition of the symbols, please refer to the notation Section at the end of this paper. The details of these formulas are given in the references: (Martinez-Rueda, 1997) and the application of these formulas are given in the references: ((Martinez-Rueda, 1997); (Konstantinidis et al., 2004)).

Models of Mander, Priestley and Park

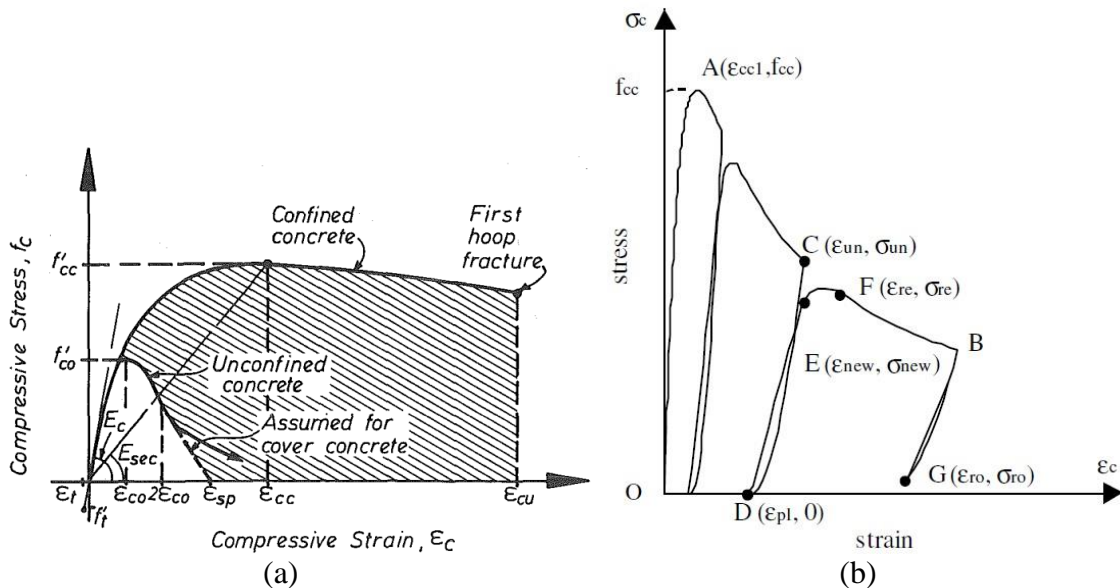


Figure 5. (a) Stress-strain model for monotonic loading (Mander et al., 1989)
 (b) Cyclic loading stress-strain diagram (Mander et al., 1989)

The study by Mander et al., (1989) proposed a technique that was utilized to show how both confined and unconfined concrete behave. A unified model for confined concrete subjected to cyclic and monotonic compressive loading. This model is a streamlined variant of the Karsan and Jirsa model shows that design the capacity of concrete to convey about tensile stresses. A stress-strain relationship is constructed using a single equation developed by the model. Due to its generalization, research and design have been presently adopted extensively by this approach. An equation by Popovics (1973) gives the envelope curve incorporating the impact of confinement. The unloading branches connect the point C (strain reversal point) to point of full reversal i.e. point D having zero slope. Reloading occurs on two branches. Strains lesser than highest strain observed a line was inserted amid point of reloading (point G) and descending point of strength (point E). Strain bigger than maximum strain joins the degrading point of strength (point E) with a parabolic curve to the point of returning (point F) along the envelope. There is a varying equation of inelastic strain when the highest strain observed is altered, this occurs on the second reloading branch or the envelope.

Envelope Curve

$$\sigma = \frac{f'_{cc}x^r}{r-1+x^r} \tag{17}$$

Where:

$$x = \frac{\epsilon_c}{\epsilon_{cc}} \tag{18}$$

$$r = \frac{E_c}{E_c - E_{cl}} \tag{19}$$

Reloading curve

$$\sigma_{new} = 0.92\sigma_{un} + 0.08\sigma_{ro} \tag{20}$$

Unloading Curve

$$\sigma = \sigma_{un} - \frac{\sigma_{un} \left(\frac{\epsilon - \epsilon_{un}}{\epsilon_{pl} + \epsilon_{un}} \right) \frac{bcE_c}{bcE_c - E_{cl}}}{\frac{bcE_c}{bcE_c - E_{cl}} - 1 + \left(\frac{\epsilon - \epsilon_{un}}{\epsilon_{pl} + \epsilon_{un}} \right) \frac{bcE_c}{bcE_c - E_{cl}}} \tag{21}$$

Unloading and Reloading Curve

$$\text{First branch: } \sigma = \sigma_{ro} + E_r(\epsilon - \epsilon_{ro}) \tag{22}$$

$$\text{Second branch: } \sigma = \sigma_{re} + E_r(\epsilon - \epsilon_{re}) + A(\epsilon - \epsilon_{re})^2 \tag{23}$$

For the definition of the symbols, please refer to the notation Section at the end of this paper. The details of these formulas are given in the references: ((Mander et al., 1989); (Karsan and Jirsa, 1969); (Popovics 1973); (Aslani and Jowkarmeimandi 2012)) and the application of these formulas are given in the references: ((Youssef and Moftah, 2007); (Konstantinidis et al., 2004)).

Models of Hognestad

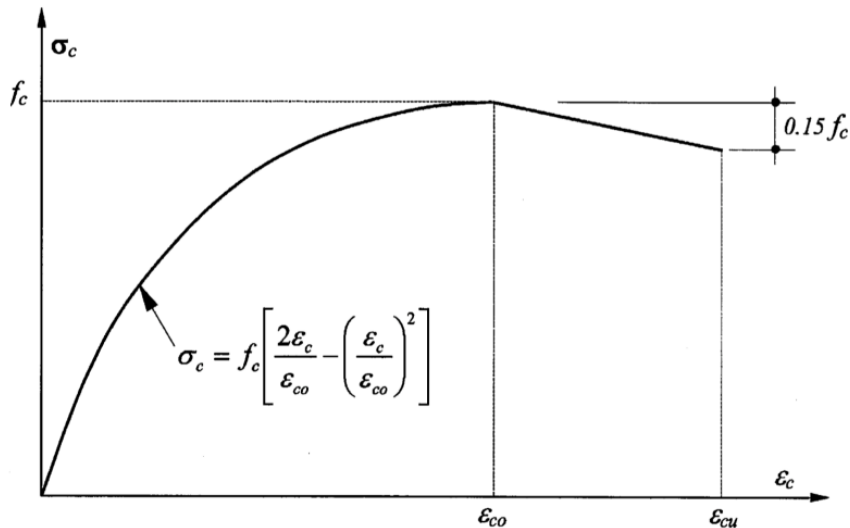


Figure 6. Stress-strain model under monotonic loading (Hognestad, 1951)

The model by Hognestad (1951) is generally being utilized for the stress-strain curve to symbolize the nature of normal strength and present it as an acceptable model for unconfined. In the model, stress-strain curve bend up to the pinnacle portion expected a second order parabola and part of the decrease is thought to be linear. The maximum stress, typically taken as 85 percent cylindrical strength of concrete i.e. $f_c = 0.85f_{ck}$ and the maximum compressive stress compared to strain (ϵ_{c0}) is taken to be 0.002.

Envelope Curve

$$\sigma_c = f'_c \left[1 - 0.15 \left(\frac{\epsilon_c - \epsilon'_c}{\epsilon_u - \epsilon'_c} \right) \right] \quad \epsilon'_c \leq \epsilon_c \leq \epsilon_u \quad (24)$$

$$\sigma_c = f'_c \left[\frac{2\epsilon_c}{\epsilon'_c} - \left(\frac{\epsilon_c}{\epsilon'_c} \right)^2 \right] \quad \epsilon_c \leq \epsilon'_c \quad (25)$$

For the definition of the symbols, please refer to the notation Section at the end of this paper. The details and applications of these formulas are given in the reference: (Hognestad (1951)).

Models of Kent and Park

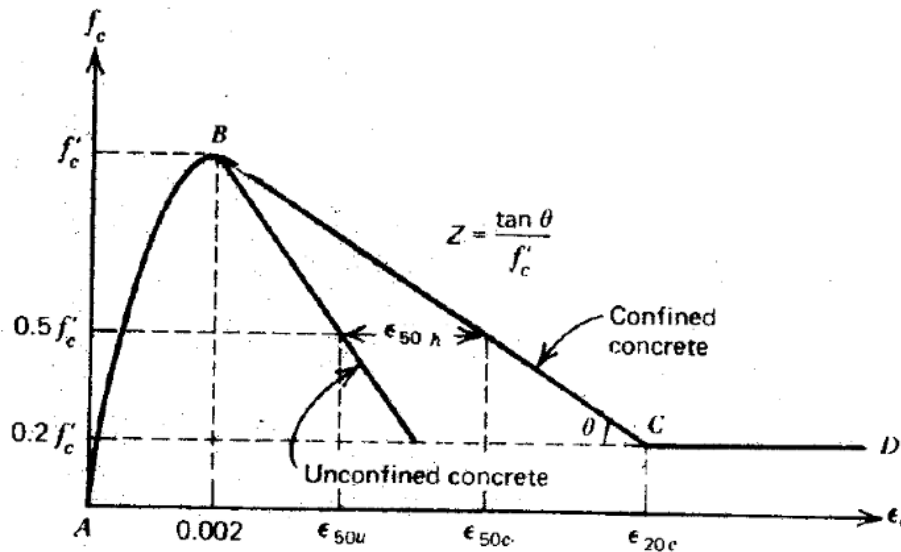


Figure 7. Stress-strain model under monotonic loading (Kent et al., 1972)

In their model, the climbing branch is symbolized by updating the Hognestad’s 2nd-degree parabola changing $f_c = f_{ck}$, furthermore the part up to the highest point of the curve.

Envelope Curve

$$\sigma_c = f_c \left[\frac{2\epsilon_c}{0.002} - \left(\frac{\epsilon_c}{0.002} \right)^2 \right] \quad (26)$$

Also, the post-peak branch is regarded as a straight line having a slope that is characterized basically as a function of the strength of concrete;

$$\sigma_c = f_c \{ 1 - Z(\epsilon_c - 0.002) \} \quad (27)$$

For the definition of the symbols, please refer to the notation Section at the end of this paper. The details of these formulas are given in the references: ((Kent et al., 1972); (Hognestad (1951)) and the application of these formulas are given in the references: ((Sadeghi and Nouban, 2010); (Carreira and Chu 1985)).

CONCLUSION

Most of the researchers after their investigations concluded that there was no significant difference between envelope curves of cyclic loading and envelope curves of monotonic loading. Some of the models are not entirely original but modified from previous models taking into account some new parameters. So far, these models have been proven and validated experimentally and analytically to be true.

Notation

E_c :	tangent elasticity modulus of concrete
E_{sec} :	secant elasticity modulus of concrete
n :	material criterion based on the shape of the stress-strain curve
ε_c :	longitudinal compressive concrete strain
ε'_c :	tensile strain
ε_{pl} :	plastic strain
ε_{ro} :	reloading concrete strain
ε_{un} :	unloading concrete strain
σ_c :	general concrete stress
σ_f :	crack closure stress
σ_{new} :	deteriorated concrete stress
σ_{ro} :	initial concrete stress on reloading branch
σ_{un} :	reversal envelope stress
f'_c :	compressive strength of concrete
f'_{cc} :	confine concrete compressive strength
ε_{cc} :	compressive strain of longitudinal confined concrete
ε_c :	longitudinal compressive concrete strain
f'_{co} :	compressive strength of unconfined concrete
ε_{co} :	compressive strain of unconfined concrete

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