THE EFFECT OF A CONSTRUCTIVIST TEACHING APPROACH ON 10TH GRADE STUDENTS' CONCEPTUAL LEARNING OF LOGARITHM FUNCTION

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ABSTRACT

The aim of the current research is to investigate the effect of constructivist approach on conceptual learning of logarithm function. This study has been administered to 26 tenth grade students (12 female, 14 male) over a 3 week-period (12 courses).At the end of the treatment period, the students were asked 10 open-ended questions designed to probe their conceptual learning of logarithm. This study is a qualitative research and the method of descriptive analysis was used to analyse the data. As a result, it was determined that most of the students (85%) failed in comprehension the logarithm function.

Keywords: constructivist approach, conceptual learning, logarithm function

INTRODUCTION

The research that was conducted on logarithm showed that students' understanding of logarithm function was quite limited and they could only understand logarithm as an action and did not understand this concept as a process (Chesler, 2006;Confrey& Smith, 1995; Weber, 2002a; Weber, 2002b ; Williams, 2011). Students generally do not have a good understanding of logarithms. Students tend to remember the rules incorrectly and use these mis-remembered rules without making sure they are correct, perhaps in part because they do not know how to check for correctness (Kastberg, 2002; Kenney, 2005). This is consistent with the opening quotation by Hiebert (2003), because he wrote that students who memorize rules and procedures without understanding cannot extend their knowledge or check for correctness.

Students struggle, greatly, with both the concept of logarithms as inverse functions and the processes and procedures needed for working with logarithmic equations. Much of this difficulty stems from trouble students have interpreting notation used to express logarithms (Kenney, 2005). While mathematics educators proposed instructional techniques to supplement or replace traditional pedagogy of exponents and logarithms (e.g., Confrey& Smith, 1995; Rahn&Berndes, 1994). Students can seldom explain why these properties are true (Weber, 2002a). However, at the end of the research by Weber (2002), the students who received the instruction that was used in the process of the research, performed better than students who received traditional instruction at performing basic computations, recalling rules, and explaining why the rules of exponents and logarithms are true. They were also better able to answer questions that required them to use their conceptual knowledge of these topics. In a study by Kenney (2005), he investigated how college students interpreted logarithmic notation and how these interpretations informed students' understandings of rules for working with logarithmic equations. The framework used for this study was the procept theory of Gray and Tall (1994). The results suggested that most students in the study lacked a process-object understanding of logarithms. The results of this study indicated that the students did not, in general, have a perceptual understanding of logarithms. The major

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problem that students seemed to have with logarithms was making meaningful connections to name of the logarithm and the notation used to represent it.

To be successful, students must be able to interpret the symbols used as both an expression of the object of a logarithm and an indication of the process needed to work with the function (Gray& Tall, 1994; Kinzel, 1999; Sajka, 2003; Weber, 2002b). To solve both exponential and logarithmic equations, students must be able to understand connections between the logarithmic and exponential forms and be able to combine and reverse the processes involved in both forms (Dubinsky &Harel, 1992; Weber, 2002a). However, Hurwitz finds that logarithmic notation leaves students "bereft of a succinct way to verbalize the operation performed on the input" (p. 344), and that the change from the familiar f(x) makes it difficult for students to interpret the logarithm as a function output. Williams (2011) concluded that for students to depend on the practice of switching forms as a primary way to deal with logarithms was not helpful. Instead, he suggested that students depended primarily upon the object and process definitions for logarithms as their primary ways to deal with logarithms. Berezovski and Zazkis (2006) split "understanding logarithms" into three categories: logarithms as numbers, logarithms as operations, and logarithms as functions. Williams (2011) believed that this framework was not complete because he found that students studied experienced difficulties with logarithms that were not explained by the framework developed by Berezovski and Zazkis (2006). He modified the framework given by Berezovski and Zazkis (2006) as explained below:

1. logarithms as objects 2. logarithms as processes 3. logarithms as functions 4. logarithms in contextual problems. Both mathematically and epistemologically, logarithms as functions differ from logarithms as numbers or operators (Smith &Confrey, 1994). From this point, Weber (2002b) suggests these 2 steps: 1. initiating a discourse on logarithms as numbers and operators, 2. initiating a discourse on logarithms as functions.

Little research in math education looked specifically at students' understanding of logarithms (Weber, 2002a; 2002b). On the other hand, any research on students' understandings when using a constructivist approach in the process of learning the function of logarithms was not encountered. The current research focused on whether constructivist approachs improve students' understandings of the function of logarithm and its rules.

METHOD

The aim of this research is to investigate the effect of constructivist approaches on conceptual learning. This study has been administered to 26 tenth grade students (12 female, 14 male) over a 3 week-period (12 courses). In Turkey, the students of this kind of school are selected according to the result of a national exam. Therefore it can be said that they are generally successful compared to the other high school students. On the other hand, the participants were the most successful students among the tenth graders in terms of mathematical ability. The courses were conducted by the researcher who has taught Math for 15 years and has a PhD in mathematics education.

In the courses, the aim was to have students discover the logarithm function and properties on the basis of exponential function and inverse function. A sample are relating to logarithm function is below.

1) Which properties have a function that is an inverse function?

2) Do exponential functions have an inverse function?

3) $f:R\rightarrow R^{+}$, $f(x)=a^{x}$, $a\in R^{+}\backslash\{1\}$. Define $f^{-1}(x)$.

Here $f^{-1}(x)$ is represented with $\log_a x$.

Definition: (the definition of the function of logarithm)

Example: Venn diagram for $f(x)= 2^x$ and $f^{-1}(x)= \log_2 x$.

Example:Venn diagram for $f(x)=(2/3)^x$ and $f^{-1}(x)$. $\log_{2/3}27/8$ =?

Question: $\log_{10}(2x)$, $\log_7(x-1/x)$, $\log_{1/2}(x+1)$, $\log_3(x-9/x -2)$ the domain of the functions?

Question: $\log_3 9$=? $\log_2 1/8$=? $\log_{1/5}25$=? $\log_6 6$=? $\log_8 1$=? $\log_2 0$=?

Question: $f(x)=(3/4)^x$ $f^{-1}(16/9)$=? How is it expressed using a logarithm?

$f(x)= 2^x$ $f^{-1}(1/16)$=? $f^{-1}(8)$=? $f^{-1}(x)= \log_2 x$ $\log_2 8=3$

Question: $y= a^x \rightarrow x$=? ($\log_a y$)

The questions relating to graphic of the logarithm function.

1) Plot $\log_2 x$ and $\log_{2/3} x$. What is the function that is symmetric according to y=x?

2) Plot the function $y=\log_a x$ depending upon "a".

3) Plot $y=\log_3(2x-1)$.

The examples relating the properties of the function of logarithm

Example: 2^{log2^3} =? 5^{log5^7} = ?

Solution 1) The teacher asked the students the following question. "What exponential function must we consider to be able to solve this question?", "How can be set a Venn diagram to solve the question?"

Solution 2) $loga^y = x \leftrightarrow y = a^x$. How can the problem be solved using this equality?

Solution 3) $f(x)=a^x$ $(fof^{-1})(x)=x$. How can the problem be solved using this equality?

Solution 4) $x=y \rightarrow loga^x = loga^y \rightarrow x = a^{loga^y} \rightarrow y = a^{loga^y}$

At the end of the treatment period, the students were asked 10 open-ended questions designed to probe their conceptual understanding of logarithms.

FINDINGS

The frequencies of the students who gave correct answer are given in Table 1.

Table 1. The Students' Responses

Question	*N, %*
1) $\log_a b = c \Leftrightarrow b = a^c$. What is the rationale of this statement?	4, 15%
2) What do you understand by logarithm?	6, 23%
3) Why isn't a logarithm defined for negative numbers?	6, 23%
4) $\log_{2^3} 5^7 = \frac{7}{3}\log_2 5$. Why?	3, 12%
5) $\log_a \frac{x}{y} = \log_a x - \log_a y$. Why?	2, 8%
6) $\log_2 \frac{1}{3} = \log_{\frac{1}{2}} 3$. Why?	4, 15%
7)Order the numbers $\log_{\frac{1}{2}} 7, \log_{\frac{1}{2}} 5, \log_2 \frac{1}{3}$	15, 58%
8) a) $\log_2 x \leq \log_3 x$ b) $\log_{\frac{2}{3}} x > \log_{\frac{3}{5}} x$. Solution?	2, 8%

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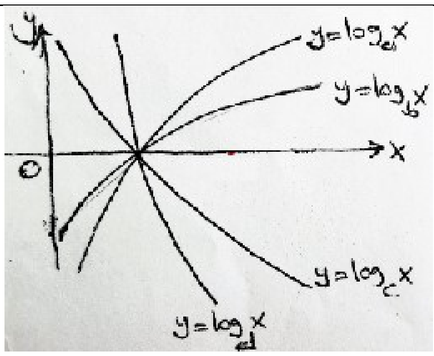
9) 	0
10) $f(x.y)=f(x)+f(y)$, $f(x/y)=f(x)-f(y)$, $f(x^n)=nf(x)$ Write a function having these properties.	2, 7%

CONCLUSION AND DISCUSSION

It was seen that on average, only 15% of the answers given by students were correct. The needed pre-knowledge was function concept and graphic literacy at the basic level. Also, the ability of using this knowledge in the process of problem solving is important as well. In this situation, when assumed that the students had basic knowledge to solve these problems, that the students were not able to transfer their knowledge into the process of problem solving can be considered an important reason of this failure.

On the other hand, when investigating the literature, it was expressed that the constructivist approach will enhance conceptual understanding. However, in this research a similar result was not obtained. Therefore, it can be said that only constructivist approach is not adequate to enhance the success and, there would be some other factors affecting learning. The only aim of the students in Turkey is to be successful in the multiple choice tests and in general these test do not contain conceptual knowledge. Therefore, the efforts to do conceptual learning do not produce positive results although constructivists approach.

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