

Dynamic Pricing by Restricted Boltzmann Machine

Lusajo M. Minga¹, Noel E. Mbonde²

Mbeya University of Science and Technology, Mbeya,
TANZANIA.

¹lusajominga.103@gmail.com, ²noelmbonde@yahoo.com

ABSTRACT

This paper presents an investigation of mechanisms for price setting algorithms on dynamic pricing based on group buying discounts for electronic commerce using Restricted Boltzmann Machine. It is proposed for the optimal cost of production functions which provide the marginal cost for the unit price of the product. The investigation finds techniques to set automatic the profitable price for online group buying discounts where the marginal cost is changing with the change of quantity demand. The simple example of production functions is used for Restricted Boltzmann Machine. The example confirmed the convergence of production costs within their isoquants. The production costs continue to decrease up to the point where they can't decrease any more. Those are the points of the optimal costs, from which we obtain the marginal cost to be inserted automatic into the formula for the calculation of the selling price of the product.

Keywords: E-commerce, Restricted Boltzmann Machine, dynamic pricing, production function

INTRODUCTION

Research on dynamic pricing is important for the development of e-commerce. Dynamic pricing has become the main characteristic of e-commerce. Most business in e-commerce do not have fixed prices they use dynamic pricing. Dynamic pricing has advantages to both sellers and buyers. Sellers get more profit, and buyers buy at low price and remain with some surpluses. Dynamic pricing is one method of price discrimination, which is the practice of charging different prices to different consumers for similar goods. Before the internet era price discrimination was expensive because sellers were using a lot of time and money to gather information about buyers.

Sometimes price discriminations could cause price war, because sellers were required to compare their prices with the price of competitors. Then they were forced to set the selling below the price of competitors to attract more buyers. With this type of price setting, sellers could even set the selling price under the production cost leading to the loss of the firm. At this time of the internet era gathering information from buyers is not expensive any more [3, 6]. Sellers can get information about buyers while sitting on their desks at the lowest cost. Therefore it is possible for sellers to set automatic the online selling price as they get information about buyers at a lowest cost. The following section is the review of unit selling price, followed by the review of Restricted Boltzmann Machine. Section four explains the dynamic pricing by using Restricted Boltzmann Machine, followed by example and analysis, the last section is a conclusion.

UNIT SELLING PRICE

The unit selling price is set by the seller. It is an amount that a seller will charge per one commodity when selling the product. When setting the unit selling price the seller has to consider about the total production cost and profit. A production of a commodity with more than one variable production factors has many different production combinations of production factors to produce one ordered quantity of a product. With different combinations of production factors there is only one combination which can provide the highest profit to the producer. The combination of production factors with highest profit is found at the point of production when the marginal revenue equals to the marginal cost. The marginal cost found at this point can be inserted into the pricing equation which is the function of the marginal cost and elasticity of demand [5, 4].

The calculation of the marginal cost to be inserted into the offline unit selling price is not difficult. The challenge comes when the seller wants to calculate the online selling price where different quantity orders are pressed at one time and different prices according to the quantity ordered are needed to be submitted at one time. This is a point where the seller will need some sort of automation for marginal cost calculations then inserting into the pricing equation for the selling unit price calculations. In this case Restricted Boltzmann Machine is used for the automation of the marginal cost calculations and the unit selling price setting automatic.

RESTRICTED BOLTZMANN MACHINES

Restricted Boltzmann machines (RBM) is an energy-based neural network [1, 7]. The network learns the probability distribution of the input stimuli through unsupervised learning. The network [8] consists of two layers of neurons, one visible layer and one hidden layer with no visible-visible or hidden-hidden connections. Visible layer is the input frame to the network, while the capability of the network in modeling the distribution of the input data is determined by the number of hidden neurons. The architecture of the RBM is illustrated in the Fig. below.

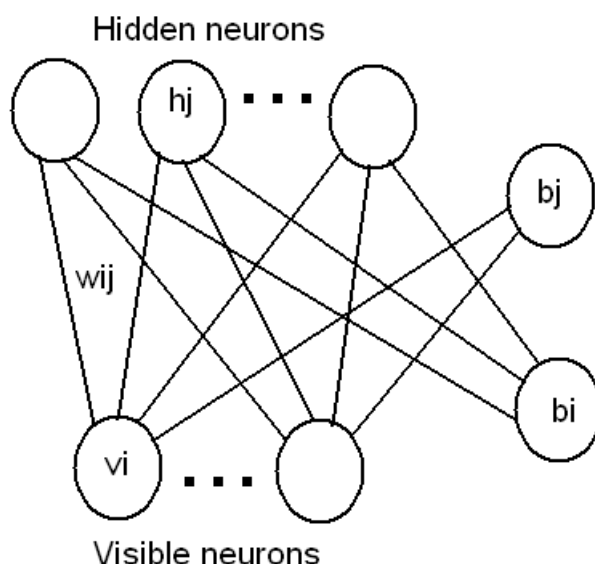


Figure1: Architecture of the RBM

On Figure1 above, the indices of the visible and hidden units are denoted by i and j , respectively. w_{ij} is the weight of the connection between the i th visible unit and the j th

hidden unit, b_i is the bias for the i th visible unit, and b_j is the bias for the j th hidden unit. The energy function of the RBM is given as follows:

$$E(v, h) = - \sum_i \sum_j v_i h_j w_{ij} - \sum_i v_i b_i - \sum_j h_j b_j \quad (1)$$

where v_i is the state of the i th visible unit and h_j is the state of the j th hidden unit.

The learning procedure does approximate gradient descent in a quantity called "contrastive divergence" [2].

Contrastive Divergence

In RBM modeling, the network is trained using contrastive divergence (CD) learning [1, 2, 10] in order to obtain the trained weights and biases. The training of the RBM undergoes two phases (positive and negative phases). During the positive phase, the conditional probability of the hidden units given the input data is computed according to an Eq. below:

$$P(h_j | v) = \frac{1}{1 + \exp(-(\sum_i w_{ij} v_i + b_j))} \quad (2)$$

Where b_j is the bias for hidden unit, and $P(h_j|v)$ is the conditional probability of h_j given v . Subsequently, the states of the hidden neurons $\langle h_j \rangle_0$ are sampled. During the negative phase, the hidden layer is stochastically fired to reconstruct the visible data $\langle v_i \rangle_1$ by sampling according to the following equation:

$$P(v_i | h) = \frac{1}{1 + \exp(-(\sum_j w_{ji} h_j + b_i))} \quad (3)$$

The hidden layer $\langle h_j \rangle_1$ is, subsequently, recomputed from the visible layer using the equation above after which, the weights and biases are updated as follows:

$$\begin{aligned} w'_{ij} &= w_{ij} + \epsilon (\langle v_i h_j \rangle_0 - \langle v_i h_j \rangle_1) \\ b'_i &= b_i + \epsilon (\langle v_i \rangle_0 - \langle v_i \rangle_1) \\ b'_j &= b_j + \epsilon (\langle h_j \rangle_0 - \langle h_j \rangle_1) \end{aligned} \quad (4)$$

$\langle \rangle_0$ denotes the states of the neurons before reconstruction, and $\langle \rangle_1$ denotes the states of the neurons after a single step reconstruction. The same procedure, including positive phase, negative phases, and weight updating, are repeated until the stopping criterion is reached. The energy value is calculated as per equation (2), the probabilistic model is constructed by considering the energy value of the network to have new data points. After the RBM learning process finishes it can be shown how well it performs on new data points

DYNAMIC PRICING BY RBM

The basic approach of solving the minimum cost of a product with a RBM is to estimate the joint probability distribution of the calculated bundle costs for each production factor. The study provides a clear picture of how the joint distribution of the bundle cost is related to the marginal distributions of the bundle cost. The joint probability distribution for bundle costs gives the probability that each of the bundle cost falls in any particular production factors, for production cost. The joint probability distribution is obtained from the marginal distribution

which gives the probabilities of various bundle costs to be used by different production factors without referring to other bundle costs.

The process starts with a wide set of bundle costs and ends up with a reduced number of those bundle costs. In CD training the training starts with a given calculated of bundle costs, then the CD training extends the bundle costs by defining new ones and then reduces the number as the different states of neurons occurs in the marginal distribution of bundle costs. Several different states are involves each state treats a different subset of bundle costs as the marginal variables.

The joint probability distribution of the selected individuals is estimated by using the algorithm which was proposed by [9, 11]. The output of the network is the marginal probability distribution of the bundle costs for each production factor. The distribution is factorized as a product of independent univariate marginal distribution of each bundle cost. Finally, sampling is performed to generate new bundle costs. The new bundle costs are used to calculate the minimum production cost, which is applied in the calculation of the unit price for the quantity ordered.

The joint probability distribution is formulated as:

$$P(v) = P(v | Pop) = \prod_{i=1}^n P(v_i) \tag{5}$$

where

$$P(v_i = 1) = \frac{P(v_i)^+ + avg(P(v_i))}{P(v_i)^+ + P(v_i)^- + r_i avg(P(v_i))}$$

$$P(v_i)^+ = \sum_{h=1}^H e^{-E(v_i=1,h)}$$

$$P(v_i)^- = \sum_{h=1}^H e^{-E(v_i=0,h)}$$

and where v_i is the bundle cost of i th production factor, n is the number of bundle cost, H is the number of hidden units, $P(v_i)^+$ is the marginal cost of v_i when the cardinality of $v_i = 1$, $P(v_i)^-$ is the marginal cost of v_i when the cardinality of $v_i = 0$, and E is the energy function of the RBM.

The new bundle costs are generated by sampling the computed probability distribution, according to the following equation:

$$v_i = \begin{cases} 1 & \text{if } random(0, 1) \leq P(v_i) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Where v_j is a newly generated bundle cost for j^{th} production factor, $random(0,1)$ is a randomly generated value between 0 and 1.

Flowchart

Flowchart of price setting algorithm for group buying discount on e-commerce by Restricted Boltzmann Machine.

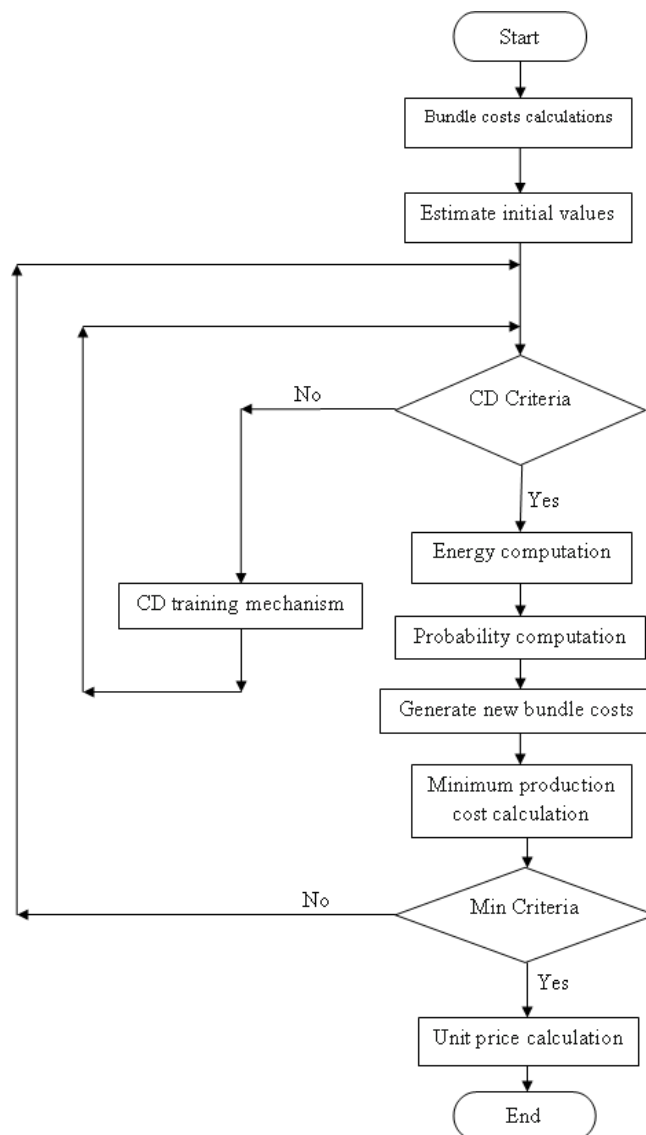


Figure2: Flowchart of price setting algorithm by RBM

In Figure2 above, the computed minimum cost, gives the marginal cost for each quantity ordered, and the marginal cost inserted automatically into the pricing formula. The algorithm sets the selling price only at the maximum profit when marginal cost equals to marginal revenue.

Simulation Analysis For Price Setting By Using RBM

The assumption is a firm that has to sell more than one product quantity at a time. The product has four input factors. The quantity to sell and pricing decisions are made simultaneously. A firm sells different levels of quantities conditional on the demand. To facilitate maximizing profit the firm will try to use a combination of inputs that will minimize its total cost of a given level of quantity demanded. Firm is employing methods dynamically adjusting price over time based on quantity demanded.

With this example of four production factors simulation was done by using Matlab 7.10.0.499 (R2010a) on a computer with clock rate 2.66GHz. The example shows us the relationship between the states obtained, marginal cost, unit price, time used, joint probability distribution (JPD) and quantity ordered.

Table 1. Simulation Results

States (Solutions)	MC	Unit Price	Time (sec.)	JPD	Quantity (q)
8669	10.38	20.77	8.6	0.0625	20
27000	10.01	20.02	36.85	0.0385	600
27000	10.00	20.00	36.45	0.0385	8000
65518	10.38	20.77	99.73	0.0625	20
205379	10.01	20.02	329.70	0.0385	600
205379	10.00	20.00	332.14	0.0385	8000

The Table 1 shows prices of different quantities for products with four production factors. Unit Price is decreasing with the increase in quantity ordered. Marginal cost is decreasing with the increase in quantity ordered. The increase in quantity ordered, increases the number of states and increases the computational time. The joint probability distribution decreases with the increase of quantity ordered.

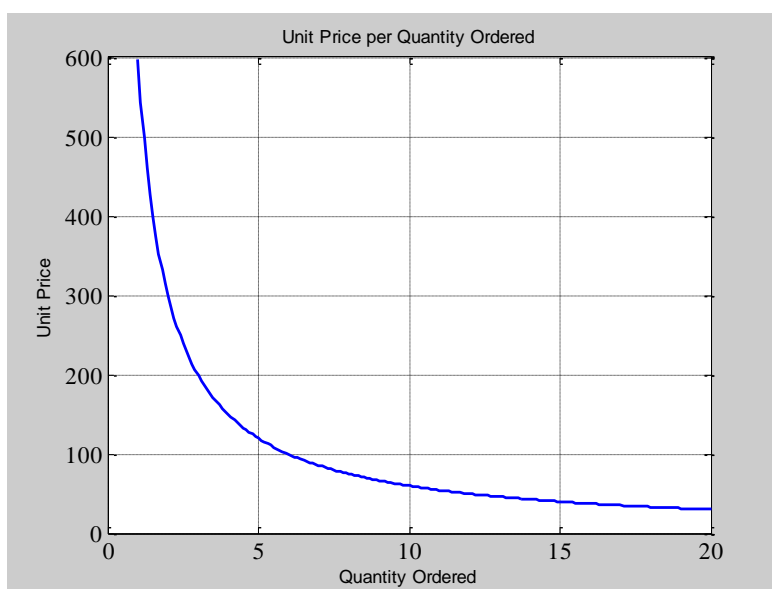


Figure3: Graph of Unit Price per Quantity Ordered

The Figure3 above of the dynamic pricing model shows the decrease of per unit price with the increase in quantity ordered. A Seller can set his first price but as quantity ordered increases, the price will continue to drop along the price curve.

CONCLUSION

Since our main objective of this investigation was to find an algorithm which can calculate the online price at the lowest price and lowest Marginal cost which means increasing the sellers profit and also increasing the buyers surplus, therefore the result on the Table above is our privilege. But with the shortcoming of this results that the increase in the number of quantity ordered exponentially increases computational time, thus our further research will continue on the automation of the online pricing by using the RBM using binary vectors and Pareto efficiency.

REFERENCES

- [1]. Hinton et al., (2005). *Learning Causally Linked Markov Random Fields; In: Artificial Intelligence and Statistics*, Barbados.
- [2]. Hinton, G. E. (2002). Training products of experts by minimizing contrastive divergence. *Neural Computation*, 14(8):1711-1800.
- [3]. http://www.ehow.com/about_5255769_dynamic-pricing.html#ixzz2jqYjeuL
- [4]. Lusajo et al., (2003). Dynamic Pricing E-Commerce Oriented Price setting algorithm, Proceedings of 2003 International Conference on Machine Learning and Cybernetics, November 02-05, 2003, Sheraton Hotel, Xian, China, Volume 2, pages 893-898 IEEE
- [5]. Lusajo et al., (2004). *Hopfield Neural Network for the Optimal Cost; Proceedings of the Third International Conference on Machine Learning and Cybernetics*, Shanghai, 26-29 August.
- [6]. Krugman, P.(2000). *What Price Fairness?*, N.Y. TIMES, Oct. 4, at A35.
- [7]. Salakhutdinov et al., (2007). Restricted Boltzmann machines for collaborative filtering; In Ghahramani, Z., editor, *Proceedings of the International Conference on Machine Learning*, 24, pages 791-798. ACM}.
- [8]. Smolensky, P. (1986). Information processing in dynamical systems: Foundations of harmony theory; In Rumelhart, D. E. and McClelland, J. L., editors, *Parallel Distributed Processing: Volume 1: Foundations*, pages 194-281. MIT Press, Cambridge, MA.
- [9]. Tang, et al., (2010). Restricted Boltzmann machine based algorithm for multi-objective optimization; In *IEEE Congress on Evolutionary Computation*, pp. 3958-3965.
- [10]. Tieleman, T. (2008). *Training restricted Boltzmann machines using approximations to the likelihood gradient*; In Machine Learning, Proceedings of the Twenty-first International Conference (ICML 2008). ACM.
- [11]. Vui et al., (2013). Multi-Objective Optimization with Estimation of Distribution Algorithm in a Noisy Environment. *Evolutionary Computation* 21(1): 149-177.