Performance Evaluation of Multiple Antenna Systems with Diversity Techniques Using Ber Analysis.

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ABSTRACT

The main thrust of this paper is to investigate the effectiveness of multiple-antenna systems in signal interference mitigation. Wireless communication is characterized by multipath propagation resulting to signal degradation. MATLAB Toolbox version 7.0 at R.F Processing was used to carry out BER Performance simulation in a Raleigh fading environment for the different antenna systems including Single-Input Single-Output used specifically for reference purpose. The model incorporates the use of Zero-Forcing (ZF) Equalizer at the receiver. Performance evaluation was done at Receiver BER Threshold of 10^{-3} and 10^{-4} . The diversity methods employed by the multiple antennas were also analysed by comparing their E_b/N_0 (Energy per bit to noise power spectral density ratio). Multiple-Input Single-Output with Transmit diversity technique was shown to have 3dB diversity gain over receive diversity technique indicating a more robust transmission method.

Keywords: BER, Multiple-antenna, Wireless communication, Zero-Forcing Equalizer

INTRODUCTION

Wireless communication provides means by which information is transmitted using radio frequency bands that may be characterized by their carrier frequencies, bandwidth, propagation and interference conditions. Over the years mobile wireless communication with the use of smart phones, internet and other wireless services have witnessed tremendous growth. As the demand for wireless services grow, the wireless systems need to be continuously developed further without any trade-offs in reliability and as well as capacity demand. One major problem capable of causing signal degradation in wireless communication is the interference condition caused by the various forms of scatters inherent in the propagation environment.

Novel techniques for interference mitigation are therefore needed to enable efficient and reliable wireless services. Hence the interest in signal quality improvement and spectral efficiency at the physical layer through the use of multiple antenna techniques.

LITERATURE REVIEW

Bliss et al., (2005) carried out a study of the environmental factors that affect Multiple-Input Multiple-Output (MIMO) capacity. They listed the factors as channel complexity, external interference and channel estimation error and discussed examples of space-time codes, including space-time low density parity-check codes and space-time turbo codes.

Optical wireless communication system (OWC) is rapidly gaining popularity as an effective means of transferring data over short distances. In response to the demand for higher transmission rates in OWC, researchers have explored the use of multiple-element optical wireless communication (MEOWC). Recently, a realization of the MIMO technique for

indoor OWC was proposed by Hranilovic and Kschischang (2004). The researchers used a 512x512-pixel LCD (Liquid Crystal Display) panel and a 154x154-pixel CCD (Charged Couple Device) camera for short range OWC link.

Diversity techniques such as Multiuser diversity (MUD) have also been studied in relation to the improvement of multiple antenna performance. Multiuser diversity is based on assigning channels to users with better channel quality to maximize the system throughput. Papers by Song et al., (2009), So et al., (2008), and Wang et al., (2008) deal with MUD performance. An opportunistic feedback protocol is proposed in Song et al., (2009), for multiuser diversity systems with proportional fair scheduling and maximum-throughput scheduling. An analytical model is also provided for evaluating the proposed feedback protocol.

In So et al., (2008), a time-slotted MIMO point-to-multipoint network is considered. The transmitter decides which receivers to serve in each slot to maximize the minimum normalized average data rate realised by each receiver. Capacity-fairness trade-off can be achieved by grouping users and using a two-step selection process. In Wang et al., (2008), a game theoretic approach is used to show that the network can enforce fairness among different users by employing a pricing policy that favours equal access probabilities. The Multiuser diversity is applicable at the network layer to enhance system performance (Agubor et al., 2013).

The Weibull power transformation expressions were used to evaluate the performance of selection combining (SC) and maximal-ratio-combining (MRC) diversity receivers in the presence of fading channels (Zafeiro, 2010). Computer simulation was used to obtain performance evaluation results which were in agreement with previously known performance evaluation results presented in Alexandropoulos et al, (2007). Similar study on multi-antenna systems in correlated weibull fading channels was also carried out with emphasis on performance analysis (Papadimitriou et al., 2009).

This paper, unlike others does not focus on performance of Multiple-Input Multiple-Output (MIMO) alone but also on other forms of multiple-antenna configurations such as Multiple-Input Single-Output (MISO), and Single-Input Multiple-Output (SIMO) with different diversity techniques. This form of diversity is applicable at the physical layer. Single-Input Single-Output (SISO) which is the traditional method of transmission using sectoral antennas is also considered.

RESEARCH OBJECTIVE

The main objective of this study is:

To evaluate the effectiveness of multiple-antenna systems with diversity techniques in combating multipath propagation effects in wireless communication.

METHOD

In the analysis of performance of multiple antenna systems dynamic radio channel measurements play an essential role (Agubor et al. 2014). To carry out this process, one way is to manufacture the antenna prototypes making this method very expensive, time-consuming and a difficult process. Other methods are available to evaluate multiple antennas in a reliable manner in which evaluations can be carried out theoretically where real time scenarios are simulated. This is the method adopted in this paper.

The approach involved modelling of MIMO channel and carrying out BER analysis using MATLAB Tool box version 7.0 at RF signal processing in a non-isotropic scattering environment.

Mimo Channel

For a system with 2Tx, 2Rx antennas and an equalizer at receiver unit as shown in Figure 1.

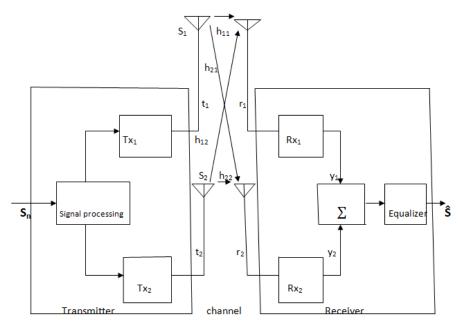


Figure 1. A simple 2 transmit and 2 receive Multiple-antenna System

The un-coded input signal S is multiplexed such that S_1 , S_2 are transmitted by antennas t_1 and t_2 respectively. The output is obtained as

$$y_1 = h_{11}S_1 + h_{12}S_2 + v_1 \dots (1)$$

$$y_1 = h_{21}S_1 + h_{22}S_2 + v_1 \dots (2)$$

Where h_{11} is the channel from t_1 to r_1

 h_{12} is the channel from t_2 to r_1

 h_{21} is the channel from t_1 to r_2

 h_{22} is the channel from t_2 to r_2

 S_1 , S_2 are the transmitted symbols

 v_1 , v_2 are the noise vectors on r_1 and r_2 respectively.

Equations 1 and 2 are represented by

$$\begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \end{bmatrix} \dots (3)$$

Similarly, for a 4-transmit and 4-receive antenna system the transmitted symbol has the sequence S_1 , S_2 , S_3 and S_4 with corresponding received signals as

$$v_1 = h_{11}S_1 + h_{12}S_2 + h_{13}S_3 + h_{14}S_4 + v_1 \dots (4)$$

$$y_2 = h_{21}S_1 + h_{22}S_2 + h_{23}S_3 + h_{24}S_4 + v_2 \dots (5)$$

$$v_3 = h_{31}S_1 + h_{32}S_2 + h_{33}S_3 + h_{34}S_4 + v_3 \dots (6)$$

$$y_4 = h_{41}S_1 + h_{42}S_2 + h_{43}S_3 + h_{44}S_4 + v_4 \dots (7)$$

In matrix notation, the equations are represented by

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \dots (8)$$

Equations 3 and 8 are similar in form. Thus, the MIMO channel is represented as

$$Y = HS + V \dots (9)$$

Where Y is the output matrix, H is the channel matrix, S is the input matrix and V is the noise matrix consisting of vectors v_1, v_2, v_3 and v_4 . In the rest of the text this will be regarded as Additive White Gaussian Noise (AWGN).

The MIMO channel model with \hat{S} as a replica of the input signal S is represented in Figure 2.

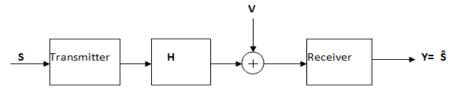


Figure 2. MIMO channel model

Description of Channel Model

- I. **Transmitter**: The transmitter chooses the waveform S_n , if n messages are to be transmitted. The set of used waveform is therefore $\{S_1, S_2, \dots, S_n\}$. The chosen waveform is input to the channel H.
- II. **AWGN:** The noise which in this text include all interfering signals and the receiver noise generated by random process get added to the received signal. The spectrum of the noise is flat for all frequencies.
- III. **Receiver:** The receiver forms the message estimate \hat{S} based on the observed waveform Y = HS + V. This is at the output of the equalizer as shown in Figure 1. The equalizer used is the Zero-Forcing Equalizer due to its effectiveness in combating Intersymbol-interference and the interest on it for *IEEE* 802.11*n* application (Mark et al., 2003).

The use of ZF equalizer should satisfy the equation (Jiang et al., 2011)

$$H^+H = I$$
(10)

Where H^+ is the pseudo-inverse of H or the equalization matrix and for a full rank matrix is defined by

$$H^{+}=(H^{H}H)^{-1}H^{H}$$
(10)

I is the identity matrix with '0' diagonal elements

Using equation 8 for an uncoded 4Tx4Rx MIMO system, the channel matrix is given by

The pseudo-inverse matrix is given by

$$H^{+} = (H^{H}H)^{-1} H^{H}$$
, hence

The matrix multiplication of equation 13, yields a diagonal matrix with non-zero diagonal elements. Therefore the equalization matrix is non diagonal, i.e

$$(\mathbf{H}^{H}\mathbf{H})^{-1} \neq \mathbf{I}$$
(14)

This results to noise amplification. Therefore the transmission method to be employed should satisfy equation 10 for the interfering signals to be nulled.

SISO with No Diversity

A case of one transmit antenna and one receive antenna (Single-Input Single-Output) is shown in Figure 3.

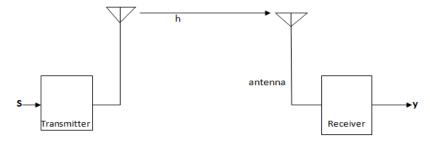


Figure 3. No-Diversity scheme(1Tx,1Rx)

The received signal takes the form:

$$v = hs + v$$
(15)

With BPSK modulation, the transmitted bits are either 1 or 0, and remain unchanged except there is a phase variation of the carrier frequency from positive to negative values. Assume signal energy transmitted is E_b then with BPSK modulation, the transmitted signals are either $+\sqrt{E_b}$ or $-\sqrt{E_b}$ for 1 or 0 bits respectively. If h is known at the receiver antenna then the instanteneous bit energy to noise ratio is

$$y_{=} |h|^2 \frac{E_b}{N_0} \dots (16)$$

Where $|h|^2$ is the channel power of a single transmit and recieve antenna.

SIMO Receive diversity technique with MRC

A receive diversity technique with maximal-ratio-combining is illustrated in Figure 4. With N_t transmitting antenna and M_r receiving antenna, Figure 4 has $N_t = 1$ and $M_r = 2$. Assume multiple M_r antennas, then for the *ith* receive antenna, the equation is similar to that of a single antenna (equation 16) and is given by

$$y_i = |h_i|^2 \frac{E_b}{N_o}$$
(17)

Where $|h_i|^2$ is the channel power across the *ith* receive antenna.

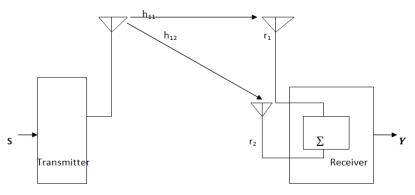


Figure 4. A simple Receive Diversity MRC scheme (1Tx, 2Rx)

Equation 13 indicates that the sum of channel power P_{ch} takes the form for M receive antennas as $|h_{11}|^2 + \dots + |h_{m1}|^2$ or

$$P_{ch} = \sum_{i=1}^{m} |h_i|^2$$
(18)

With MRC, the effective E_b/N_o at the output using equation 18 is given by

$$\gamma = \sum_{i=1}^{m} |h_i|^2 \frac{E_b}{N_0}$$
(19)

Equation 19 can be written as

$$\gamma = M\gamma_i$$
(20)

Where γ_i is the energy per bit to noise power spectral density ratio of the *ith* receive antenna (equation 17) and M the total number of receive antennas.

MISO Transmit Diversity with Alamouti STBC

A simple MISO system with $N_t = 2$ and $M_r = 1$ which employs Alamouti STBC (Space Time Block Code) is illustrated in Figure 5. The scheme has an input vector S with a transmission sequence (S_1, S_2) transmitted in two symbol periods t and t + T. The symbols are grouped into S_1S_2 and $-S_2^*S_1^*$ respectively, where * is the complex conjugate operation (Alamouti 1998).

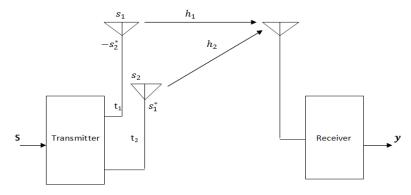


Figure 5. Transmit Diversity Alamouti STBC scheme (2Tx, 1Rx)

The encoding sequence is as indicated in Table 1 (Agubor et al., 2014), and the detected symbols $\hat{S}_1\hat{S}_2$ in equation 23.

Table 1. Channel Parameters for Transmit diversity of Alamouti STBC

Period	Transmit Antenna 1 and 2	Receive Antenna
t	S_1 S_2	h_1S_1 h_2S_2
t + T	$-S_2^*$ S_1^*	$-h_1S_2^* h_2S_1^*$

At time slot t and t + T the received signals are:

$$y_{1} = h_{1}S_{1} + h_{2}S_{2} + v_{1}$$
(21)

$$y_{2} = h_{2}S_{1}^{*} - h_{1}S_{2}^{*} + v_{2}$$
(22)

$$\begin{bmatrix} \hat{S}_{1} \\ \hat{S}_{2} \end{bmatrix} = \begin{bmatrix} S_{1} \\ S_{2} \end{bmatrix} + (H^{H}H)^{-1}H^{H} \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix}$$
(23)

Equation 23 indicates \hat{S}_1 , \hat{S}_2 as the recovered signal of the transmitted symbols S_1 , S_2 . The noise terms are nulled because the equalization matrix is an identity matrix and so will have no serious effect on the detection of the symbols.

MIMO Second-order diversity with STBC

A simple $N_t = M_r = 2$ antenna system is used to determine the diversity order with the application of Alamouti STBC and is illustrated in Figure 6.

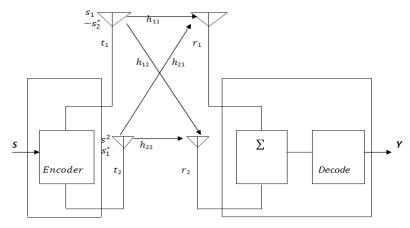


Figure 6. Second-Order Diversity with STBC (2Tx, 2Rx)

The encoding pattern is shown in Table 2.

Table 2. STBC encoding sequence of two symbol periods

Period	Transmit Antenna 1	Transmit Antenna 2
t	\mathcal{S}_1	S_2
t + T	$-S_2^*$	\mathcal{S}_1^*

The channel parameters as shown in Table 3 are used to define the signal paths between the transmit and receive antennas.

Table 3. Channel parameters for second-order diversity

Period	Transmit Antenna 1	Transmit Antenna 2
S_1, S_2	$h_{11}S_1, h_{12}S_2$	$h_{22}S_2$, $h_{21}S_1$
$-S_2^*, S_1^*$	$-h_{11}S_2^*-h_{11}S_2^*$	$h_{22}S_1^*$, $-h_{21}S_2^*$

Table 4 defines the received outputs at each given period from the two receivers.

Table 4. Signal outputs at t and t + T

Period	Transmit Antenna 1	Transmit Antenna 2	
t	y_1	y_2	
t + T	y_3	y_4	

The signals appearing at the output at each period are given by

$$y_{1} = h_{11}S_{1} + h_{12}S_{2} + v_{1}.....(24)$$

$$y_{2} = h_{21}S_{1} + h_{22}S_{2} + v_{2}....(25)$$

$$y_{3} = h_{12}S_{1}^{*} - h_{11}S_{2}^{*} + v_{3}....(26)$$

$$y_{4} = h_{22}S_{1}^{*} - h_{21}S_{2}^{*} + v_{4}....(27)$$

The recovered symbols after the equalization process are similar to that obtained in equation 23 and is given by

$$\begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + (H^H H)^{-1} H^H H \begin{bmatrix} v_1 \\ v_2 \\ v_3^* \\ v_4^* \end{bmatrix} \dots (28)$$

Equation 28 for second order diversity (2Tx2Rx) is identical to that obtained using transmit diversity technique with STBC (2Tx1Rx) in equation 23. Therefore, the BER of second-order diversity (2Tx2Rx) MIMO should be the same with that of Alamouti STBC (2Tx1Rx).

SIMULATION

The simulation was designed to carry out specific steps as illustrated in Figure 7. At the end of all the sweeps, the BER of each diversity technique was calculated and the corresponding graphs plotted. The transmit power for each antenna is assumed to be the same. The Matlab script for the simulation used the parameters tabulated in Table 5.

Table 5. System parameters

Parameter	Value/Symbol		
Transmit Antenna	N		
Receive Antenna	M		
Maximum N	2 (simulation 1); 4 (simulation 2)		
Maximum M	2 (simulation 1); 4 (simulation 2)		
E_b/N_O range in dB	0-20 in steps of 2 dB; 0-25 in steps of 5 dB		
Modulation Technique	BPSK		
Modulation Order	2		
Alamouti STBC	$S_1, -S_2^*, S_2, S_1^*$		
Channel Matrix	<u>H</u>		
System Model	$y = IhI^{2} \frac{E_{b}}{N_{o}}, y = My_{i}, \begin{bmatrix} \hat{S}_{1} \\ \hat{S}_{2} \end{bmatrix} = \begin{bmatrix} S_{1} \\ S_{2} \end{bmatrix} + (H^{H}H)^{-1}H^{H} \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix},$		

$$\begin{bmatrix} \hat{\mathbf{S}}_1 \\ \hat{\mathbf{S}}_2 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + (H^H H)^{-1} H^H \begin{bmatrix} v_1 \\ v_2 \\ v_3^* \\ v_4^* \end{bmatrix}$$

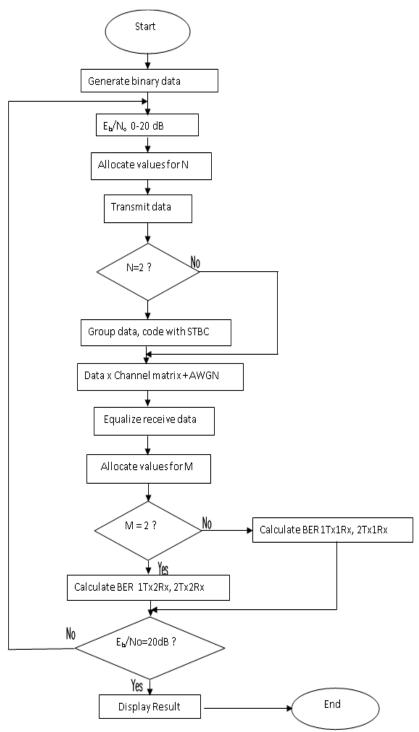


Figure 7. Simulation Flow Chart

Modulation Scheme

Modulation scheme is one of the several parameters that determine the diversity gain of a communication system. Figure 8 is a plot of BER performance of BPSK modulation scheme (Agubor et al., 2014) with No-diversity (SISO), Receive diversity (SIMO), Transmit diversity Alamouti STBC (MISO) and Second-order diversity (MIMO).

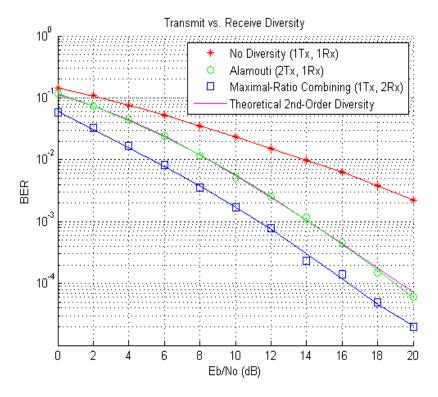


Figure 8. Diversity gain for 1Tx, 1Rx; 1Tx, 2Rx; 2Tx,1Rx; and 2Tx,2Rx antennas

The simulation result is shown in Figure 8. It has BER on the y-axis and E_b/N_0 on the x-axis. The graph of Alamouti (2Tx, 1Rx) coincides with that 2_{nd} -Order diversity. This is due to the similarity of equation 23 and 28. The BER performance at $BER = 10^{-3}$ and 10^{-4} are tabulated in Tables 6 and 7 respectively.

Table 6. Performance at BER of 10⁻³

S.No	$BER = 10^{-3}$				
	mxn	Diversity	E_b/N_O	Observation	
1	1Tx1Rx SISO	None	20	Above 10^{-3} BER at the highest E_b/N_O . of 20dB	
2	1Tx1Rx SIMO	Receive	11	Lowest power than others, improved performance	
3	2Tx1Rx MISO	Transmit	14	There is a 3dB difference with 1Tx,2Rx (i.e. 14-11=3dB)	
4	2Tx2Rx MIMO	2 nd -order	14	as above	

The performance gain of MISO and MIMO is 3dB higher than that of SIMO. In Table 7, at BER of 10^{-4} , the E_b/N_O for Receive diversity (SIMO) is 16dB; Transmit diversity (MISO) is 18.6 dB nd second-order diversity (MIMO) is 18.7dB. There is also an approximate 3dB difference between Receive diversity and the other two forms of diversity.

Table 7. Performance at BER of 10⁻⁴

S.No	$BER = 10^{-4}$				
	mxn	Diversity	E_b/N_O	Observation	
1	1Tx1Rx SISO	None	None	None	
2	1Tx2Rx SIMO	Receive	16	Lowest power than others, improved performance	
3	2Tx1Rx MISO	Transmit	18.6	Approx. 3dB difference with 1Tx,2Rx (i.e. 18.6-16=2.6dB)	
4	2Tx2Rx MIMO	2 nd -order	18.7	As above (ie 18.7- 16=2.7dB)	

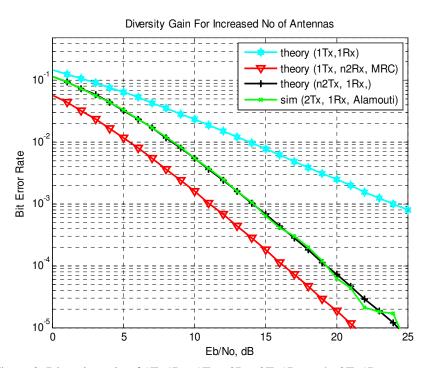


Figure 9. Diversity gain of 1Tx1Rx, 1Tx n2Rx, 2Tx1Rx and n2Tx1Rx antenna

A plot of Diversity gain for (n = 2) increased number of transmit and receive antennas is shown in Figure 9. As in Figure 6, the graph of SISO (1Tx, 1Rx) is also plotted for effective evaluation.

Tables 8 and 9 contain the BER performance at BER = 10^{-3} and 10^{-4} respectively.

Table 8. Performance at BER of 10⁻³

S.No	$BER = 10^{-4}$				
	mxn	Diversity	E_b/N_O (dB)	Observation	
1	1Tx1Rx SISO	None	24	High E_b/N_O compared to diversity systems	
2	1Tx2Rx SIMO	Receive	11	Lowest power than others, improved performance	
3	2Tx1Rx MISO	Transmit	14	A 3dB difference over Receive diversity	
4	2Tx2Rx MIMO	2 nd –order	14	As above	

Table 9. Performance at BER of 10⁻⁴

S.No	$BER = 10^{-4}$				
	mxn	Diversity	$E_b/N_O \ (dB)$	Observation	
1	1Tx1Rx	None	None	None	
2	1Tx2Rx	Receive	16.5	Lowest E_b/N_O	
3	2Tx1Rx	Transmit	19	2.5dB difference over receive diversity (approx. 3dB)	
4	2Tx2Rx	2 nd -order	19	As above	

Tables 8 and 9 show performance levels at BER of 10^{-3} and 10^{-4} respectively for n number of antennas for both transmit and receive diversity. Both tables indicate that a simple Transmit diversity system with 2Tx (transmit) and 1Rx (receive) antennas will still have the same performance even when the number of transmit antenna is increased from 2 to n number of antennas.

Power Requirements

The 3dB difference in E_b/N_O Transmit and Second-order diversity techniques have over Receive diversity (Tables 6, 7, 8 and 9) is due to the assumed power requirement. The assumption allowed each transmit antenna to have the same transmit power. The implication of this is that the systems with two transmit antennas will have double the power radiated by the system with one transmit antenna. That is Transmit and Second-order diversity techniques employing two transmit antennas each will be transmitting at a power that is twice or double that being transmitted by Receive diversity having one transmit antenna only.

Therefore, the energy radiated by the 2Tx, 1Rx and 2Tx,2Rx schemes is twice of that radiated by the 1Tx,2Rx scheme. This can be obtained as:

$$P_{rad} = 10log_{10} \ 2 = 3dB \dots (29)$$

Hence, the 3dB difference in the error performance for the receive diversity is due to the fact that its total radiated power is half the power radiated by the other diversity methods based on the simulation assumption.

CONCLUSION

It is shown in this paper using BER comparison from the simulation results that the application of multiple-antennas as against single antenna has the capabilty to operate with minimal error. Hence better performance in the face of multipath propagation effects of importance is the 3dB diversity gain using Transmit Diversity technique. This is an improvement on systems operating on Receive Diversity and those without diversity techniques (SISO). Therefore, multiple-antenna systems employing Transmit Diversity technique is a more robust transmission method. The 3dB advantage over other systems can translate to stronger signal strength with high resistance to interfering signals.

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