INELASTIC BUCKLING ANALYSIS OF AXIALLY COMPRESSED THIN CCCC PLATES USING TAYLOR-MACLAURIN DISPLACEMENT FUNCTION

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ABSTRACT

This paper is concerned with the use of Taylor-Maclaurin series in the inelastic buckling analysis of a thin, flat, rectangular, isotropic plate bounded by four clamped edges. In the problem definition, the plate is subjected to uniform uniaxial in-plane compression. The inelastic buckling behavior of the plate was obtained by adopting the deformation plasticity theory using Stowell's approach. The total potential energy functional was minimized and the inelastic buckling load was obtained using a work technique. The Taylor-Maclaurin series was truncated at the fifth term which satisfied the boundary conditions and resulted to a particular displacement function of the CCCC plate. The displacement function was substituted in the inelastic buckling equation. The critical buckling load was found to be a function of the plate buckling coefficient, and values of the plate buckling coefficient were calculated for aspect ratios ranging from 0.1 to 2.0 at intervals of 0.1. The results were compared with the elastic buckling values and the minimum and maximum percentage differences were −0.366% and −7.447% respectively. These differences show that the technique from the present study is a good approximate method for analyzing the inelastic buckling of CCCC plates.

Keywords: Boundary conditions, plastic buckling, rectangular plates, shape function, Stowell's theory, Taylor-Maclaurin series

INTRODUCTION

Buckling may be described as a condition which occurs when the equilibrium state of a structural member changes from stable to neutral under the action of a critical load. Thin rectangular plate elements used in engineering structures are often subjected to buckling through the action of axial compressive loads. Hence, it is important for the critical buckling loads and stresses to be accurately predicted. Buckling of plates may be classified into elastic buckling and inelastic (or plastic) buckling. In elastic buckling, the analysis is based on Hooke's law where it is assumed that the proportional limit of the plate material is greater than the buckling stress. In inelastic buckling, however, Hooke's law is inapplicable because of the nonlinear stress-strain relationship, and this occurs when the plate is stressed beyond the proportional limit. In many real problems, buckling may occur in the inelastic range. A number of plasticity theories have been propounded to consider the inelastic behavior, but the two main theories of plasticity used in the inelastic buckling analysis of plates are the incremental/flow theory and the deformation theory. The incremental theory was pioneered by Handelman and Prager (1948) while the deformation theory was developed by Ilyushin (1947). The deformation theory of plasticity is often preferred by researchers in the inelastic buckling of plates because its solutions are in closer agreement with experimental values, despite its weak mathematical formulation.

Inelastic buckling analyses of thin rectangular plates for various loading and boundary conditions have been investigated by researchers (Stowell, 1948; Iyengar, 1988; Shen, 1990; Wang, et al., 2004). These studies used both the deformation and incremental theories of plasticity. In terms of analytical approach, the use of numerical methods and energy methods seem to be predominant. It may be noted that no matter the method or plasticity theory used, most researchers applied Fourier series or trigonometric series as the displacement function of the deformed plate. To the best of the researchers' knowledge, none of the existing solutions from past works used the Taylor-Maclaurin series in formulating the displacement function.

Very few investigators have used Taylor-Maclaurin series in analyzing CCCC thin plate problems. Ibearugulem and Ezeh (2013) used the Taylor-Maclaurin series in formulating the shape function for the elastic stability of axially compressed CCCC thin rectangular plates. Njoku, et al. (2013) analyzed the free vibration of thin rectangular isotropic CCCC plates in the Galerkin's method using Taylor's series. Since the use of Taylor-Maclaurin series displacement function has not received much attention in literature, this present study presents a solution to the inelastic buckling of a thin, rectangular, isotropic plate for the CCCC boundary conditions. The governing equation was based on Stowell's approach and a work technique. The results were presented and compared with the elastic buckling values.

PROBLEM DEFINITION

Consider a flat, homogenous, rectangular, isotropic plate and assume that the thickness of the plate is small as compared to the other characteristic dimensions in the x- and y-directions. The thin rectangular plate is clamped along the four edges and is subjected to uniform in-plane compressive loads in the x-direction as shown in Figure 1.

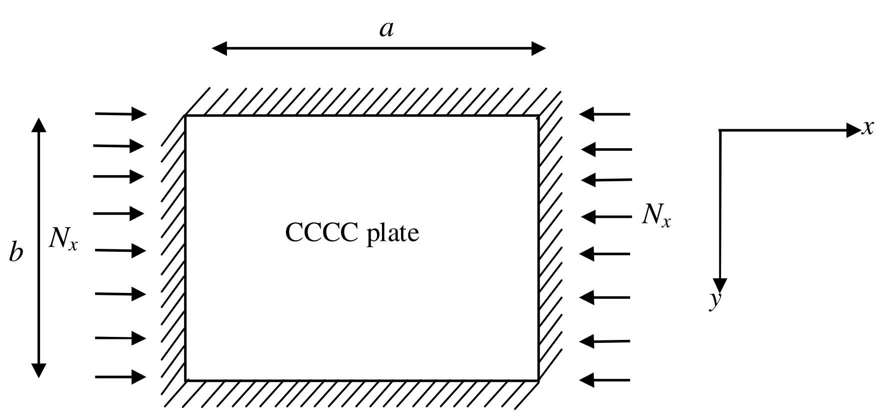

Figure 1. CCCC plate subjected to uniform in-plane compression in Cartesian coordinates

To facilitate the solution of the problem, the Cartesian coordinates are non-dimensionalized and expressed as

$$R = \frac{x}{a}; \;\; Q = \frac{y}{b} \tag{1}$$

The deflection and the slope varnish along the clamped edges. Thus, the boundary conditions of the CCCC plate are

$$w(R = 0) = 0; \;\; w'^{R}(R = 0) = 0 \tag{2}$$

$$w(R = 1) = 0; \;\; w'^{R}(R = 1) = 0 \tag{3}$$

$$w(Q=0)=0;\ \ w'^{Q}(Q=0)=0 \tag{4}$$

$$w(Q=1)=0;\ \ w'^{Q}(Q=1)=0 \tag{5}$$

Where w'^{R} and w'^{Q} are the first derivatives of the displacement functions in the R and Q directions respectively.

FORMULATIONS

Inelastic Governing Equation

Stowell (1948) derived the differential equation of equilibrium for the inelastic buckling of a thin, flat, rectangular plate subjected to uniform axial compression in the x-axis as

$$\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{t\sigma_x}{\overline{D}}\frac{\partial^2 w}{\partial x^2} \tag{6}$$

Where E_t is the tangent modulus, E_s is the secant modulus, w is the displacement function in the transverse direction, t is the thickness of the plate, $\overline{D}$ is flexural rigidity of the plate in the inelastic region and σ_x is the buckling stress. The inelastic flexural rigidity, $\overline{D}$ and the buckling load, N_x are respectively expressed as

$$\overline{D}=\frac{E_s t^3}{9} \tag{7}$$

$$N_x = t\sigma_x \tag{8}$$

Expressing Equation (6) in terms of non-dimensional coordinates with respect to Equation (8), we have

$$\frac{1}{p^4}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial R^4}+\frac{2}{p^2}\frac{\partial^4 w}{\partial R^2 \partial Q^2}+\frac{\partial^4 w}{\partial Q^4}-\frac{N_x b^2}{\overline{D}p^2}\frac{\partial^2 w}{\partial R^2}=0 \tag{9}$$

$$p = {}^{a}/_{b} \tag{10}$$

In Equations (9) and (10), p is the aspect ratio, a is the plate dimension in the R-direction and b is the plate dimension in the Q-direction.

Eziefula (2013) applied a technique based on Ibearugbulem, et al. (2013) where Equation (9) was transformed using the principle of conservation of work in a static continuum. He made N_x the subject of formula and obtained

$$N_x=\frac{\overline{D}p^2\int_0^1\int_0^1\left[\frac{1}{p^4}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{H\partial^4 H}{\partial R^4}+\frac{2}{p^2}\frac{H\partial^4 H}{\partial R^2\partial Q^2}+\frac{H\partial^4 H}{\partial Q^4}\right]\partial R\partial Q}{b^2\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\,\partial R\partial Q} \tag{11}$$

Where

$$w = AH \tag{12}$$

In Equation (12), H is the buckling curve expression and A is the amplitude of the displacement function.

Taylor-Maclaurin Series Displacement Function

Ibearugbulem (2012) expanded the displacement function using Taylor-Maclaurin series and he assumed the displacement function to be continuous and differentiable. He truncated the infinite power series at $m = n = 4$ and obtained

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} J_m K_n R^m Q^n \tag{13}$$

The boundary conditions in Equations (2), (3), (4) and (5) are now applied in Equation (13). Substituting Equations (2) and (4) into Equation (13) gave

$$J_0 = J_1 = 0;\ K_0 = K_1 = 0$$

Substituting Equation (3) into Equation (13) and solving the two resulting simultaneous equations gave

$$J_2 = J_4;\ J_3 = -2J_4$$

Again, substituting Equation (5) into Equation (13) and solving the simultaneous equations gave

$$K_2 = K_4;\ K_3 = -2K_4$$

Substituting the values of J_0, J_1, J_2, J_3, J_4, K_0, K_1, K_2, K_3 and K_4 into Equation (13) gave the unique displacement function of the CCCC plate as

$$w = \mathrm{J_4K_4}[(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)] \tag{14}$$

From Equations (12), (13) and (14), we have

$$A = \mathrm{J_4K_4} \tag{15}$$

$$H = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \tag{16}$$

Application of a Work Principle

Partial derivatives of Equation (16) with respect to *R*, *Q* or both *R* and *Q* gave

$$H\frac{\partial^4 H}{\partial R^4} = 24(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)^2 \tag{17}$$

$$H\frac{\partial^4 H}{\partial Q^4} = 24(R^2 - 2R^3 + R^4)^2(Q^2 - 2Q^3 + Q^4) \tag{18}$$

$$H\frac{\partial^4 H}{\partial R^2 \partial Q^2} = 4(1 - 6R + 6R^2)(1 - 6Q + 6Q^2)(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \tag{19}$$

$$H\frac{\partial^2 H}{\partial R^2} = 2(1 - 6R + 6R^2)(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)^2 \tag{20}$$

Equations (17), (18), (19) and (20) were expanded and integrated partially with respect to R and Q respectively in a closed domain. The results were

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial R^4}\partial R \partial Q = 0.0012698 \tag{21}$$

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q = 0.0012698 \qquad (22)$$

$$\int_0^1\int_0^1 2H\frac{\partial^4 H}{\partial R^2\partial Q^2}\partial R\partial Q = 0.00072562 \qquad (23)$$

$$\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q = 0.000030234 \qquad (24)$$

Substituting Equations (21), (22), (23) and (24) into Equation (11) gave

$$N_x = \frac{\frac{\overline{D}}{b^2}\left[\frac{0.0012698}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+0.00072562+0.0012698p^2\right]}{0.000030234} \qquad (25)$$

The inelastic buckling equation of the plate may be expressed in the form

$$\mathrm{N_x} = \frac{\pi^2\overline{D}}{b^2}k \qquad (26)$$

k is the plate buckling coefficient. Expressing Equation (25) in form Equation (26) gave

$$N_x = \frac{\pi^2\overline{D}}{b^2}\left[\frac{4.25540}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.43172+4.25540p^2\right] \qquad (27)$$

Where

$$k = \left[\frac{4.25540}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.43172+4.25540p^2\right] \qquad (28)$$

RESULTS AND DISCUSSION

The results from this study gave the equation of critical plastic buckling load as

$$\mathrm{N_{x,CR}} = \frac{\pi^2\overline{D}}{b^2}\left[\frac{4.25540}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.43172+4.25540p^2\right] \qquad (29)$$

Ibearugbulem (2012) gave the solution for the elastic stability of a CCCC thin rectangular flat plate as

$$\mathrm{N_{x,CR}} = \frac{\pi^2 D}{b^2}\left[\frac{4.255}{p^2}+\ 2.428+4.255p^2\right] \qquad (30)$$

Both solutions are upper bound, approximate solutions based on Taylor-Maclaurin series displacement function. In the present solution using Stowell's approach, $\overline{D}$ is used instead of *D*, where *D* is the elastic flexural rigidity of the plate. It may be noted that $\overline{D}$ is a function of E_s while *D* is a function of Young's modulus, *E*. In calculating the values of E_s and E_t, a comprehensive knowledge of the stress-strain curve of the plate material in the inelastic range is required. The factor E_t/E_s is equal to unity in elastic buckling but its value is always less than one in inelastic buckling. In this paper, the numerical value of E_t/E_s is taken to be equal to 0.9. Values of the plate buckling coefficient from the present study and Ibearugbulem (2012) for different aspect ratios using $E_t/E_s = 0.9$ are shown in Table 1.

Table 1. Values of *k* for uniaxially compressed CCCC thin rectangular plate

p = a/b	*k from Present Study*	*k from Ibearugbulem (2012)*	*Percentage Difference*
0.1	396.099	427.970	−7.447
0.2	101.008	108.972	−7.308
0.3	46.551	50.089	−7.063
0.4	27.714	29.703	−6.696
0.5	19.241	20.512	−6.196
0.6	14.898	15.779	−5.583
0.7	12.550	13.197	−4.902
0.8	11.306	11.800	−4.186
0.9	10.738	11.128	−3.505
1.0	10.623	10.938	−2.880
1.1	10.834	11.093	−2.335
1.2	11.293	11.510	−1.885
1.3	11.952	12.137	−1.524
1.4	12.781	12.939	−1.221
1.5	13.756	13.893	−0.986
1.6	14.862	14.983	−0.808
1.7	16.092	16.197	−0.648
1.8	17.434	17.527	−0.531
1.9	18.884	18.967	−0.438
2.0	20.437	20.512	−0.366

From Table 1, the average percentage difference is −3.325%. Comparing the solutions from the present study with those from Ibearugbulem (2012), it is noted that the closeness of the two solutions improves as the aspect ratio increases from 0.1 to 2.0. Solutions from Ibearugbulem compared favourably with Iyengar (1988) and the average percentage difference was 3.538% for aspect ratios 0.1 to 1.0 at increments of 0.1 as cited in Ibearugbulem (2012). These differences are quite acceptable in statistics as being close. Therefore, the technique from the present study is a good approximate method for estimating the displacement function in the inelastic buckling analysis of thin rectangular CCCC plates.

REFERENCES

[1] Eziefula, U. G. (2013). *Plastic Buckling Analysis of Thin Rectangular Isotropic Plates*. M.Eng Thesis Submitted to Postgraduate School, Federal University of Technology, Owerri.

[2] Handelman, G. H. & Prager, W. (1948). Plastic Buckling of Rectangular Plates under Edge Thrusts. *NACA Technical Report, 946*, 479-506.

[3] Ibearugbulem, O. M. (2012). *Application of a Direct Variational Principle in Elastic Stability Analysis of Thin Rectangular Flat Plates*. Ph.D Dissertation, Federal University of Technology, Owerri.

[4] Ibearugbulem, O. M., Ettu, L. O. & Ezeh, J. C. (2013). Direct Integration and Work Principle as New Approach in Bending Analyses of Isotropic Rectangular Plates. *The International Journal of Engineering and Science, 2*(3), 28-36.

[5] Ibearugbulem, O. M. & Ezeh, J. C. (2013). Instability of Axially Compressed CCCC Thin Rectangular Plate using Taylor-Maclaurin's Series Shape Function on Ritz Method. *Academic Research International, 4*(1), 346-351

[6] Ilyushin, A. A. (1947). The Elasto-plastic Stability of Plates. *NACA Technical Memorandum*, 1188, 1-30.

[7] Iyengar, N. G. R. (1988). *Structural Stability of Columns and Plates*. Chichester: Ellis Horwood.

[8] Njoku, K. O., Ezeh, J. C., Ibearugbulem, O. M., Ettu, L. O. & Anyaogu, L. (2013). Free Vibration Analysis of Thin Rectangular Isotropic CCCC Plate using Taylor Series Formulated Shape Function in Galerkin's Method. *Academic Research International, 4(*4), 126-132.

[9] Shen, H. S. (1990). Elasto-plastic Analysis for the Buckling and Postbuckling of Rectangular Plates under Uniaxial Compression. *Applied Mathematics and Mechanics (English Edition), 11*(10), 931-939.

[10] Stowell, E. Z. (1948). A Unified Theory of the Plastic Buckling of Columns and Plates. *NACA Technical Report, 898*, 127-137.

[11] Wang, C. M., Chen, Y. & Xiang, Y. (2004). Plastic Bucking of Rectangular Plates Subjected to Intermediate and End In-plane Loads. *International Journal of Solids and Structures, 41*, 4279-4297.