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FREE-VIBRATION ANALYSIS OF THIN RECTANGULAR FLAT PLATES USING ORDINARY FINITE DIFFERENCE METHOD

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ABSTRACT

Earlier solutions on pure bending of thin rectangular flat plates were based on the use of trigonometrical series. However, series method has the problem of improper representation of deformed shape of plate in vibration. For such problems, the use of numerical methods for approximate solutions becomes necessary. In this study, Ordinary Finite Difference method (OFDM) as one of the popular numerical techniques was used in free vibration (FB) analysis of thin rectangular flat plate. The differential equations of the biharmonic plate were transformed to fit the chosen grid pattern and these transformed equations were expressed in finite difference (FD) form. These differences were evaluated at each nodal point to obtain a set of simultaneous algebraic equations that were solved for the unknown functional values after using the proper boundary conditions of SSSS, CCCC and CSCS respectively. Visual Basic (VB) software program was developed and used in solving the algebraic equations, while the resulting natural frequencies ω, were compared with the exact values for an aspect ratio of 1.0 as shown in tables 1, 2 and 3 respectively. The solutions obtained in this study approximated closely to the exact solutions as shown in the tables. Hence, ordinary finite difference method (OFDM) provides simple and approximate solutions that are very close to exact values for this family of problems.

Keywords: Boundary Conditions, Characteristic Equation, Eigen values and Eigenvectors, Fundamental Natural Frequency, Ordinary Finite Difference Method, Resonance

INTRODUCTION

Free-vibration analysis of thin rectangular flat plates is of interest in the field of mechanics, civil and aerospace engineering. In the past years, plate problems have been treated by the use of Fourier series or trigonometric series as the shape function of the deformed plate. However, no matter the approach used, the use of trigonometric series (double Fourier series and single Fourier series) has been predominant. Most times, when it is becoming intractable to use the trigonometric series, trial and error means of getting a shape function that would approximate the deformed shape of the plate would be used (Ibearugbulem, et al.).

Gorman (1982) carried out free-vibration analysis of SSSS and CSCS plates using single Fourier series. Ventsel and Krauthammer (2001) used the Galerkin's method to carry out free-vibration analysis of SSSS plate. Szilard (2004) used ordinary finite difference method to determine the fundamental natural frequency of a square plate with all the sides clamped (CCCC). Jiu, et al. (2007) used Bessel functions to carry out free vibration analysis of SSSS, CCCC and CSCS plates. Mansour, et al. (2008) presented an analytical solution for free vibration of four edged simply supported **(SSSS)** rectangular Kirchhoff plate by wave propagation method.

In this paper, the ordinary finite difference method (OFDM) was used to obtain solutions for the free-vibration analysis of thin rectangular flat plates carrying uniformly distributed load with the following boundary conditions (Figures 1 - 3): (i) SSSS; (ii) CCCC and (iii) CSCS. An interactive ordinary finite difference method based software program was written in Visual Basic and provided to make the solution easy.

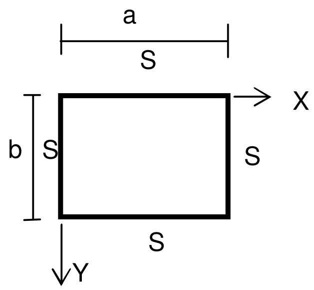

Figure 1: SSSS plate

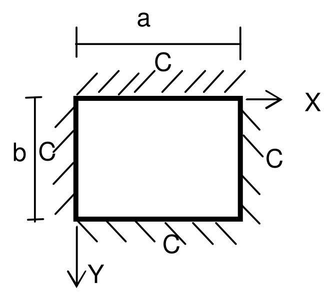

Figure 2: CCCC plate

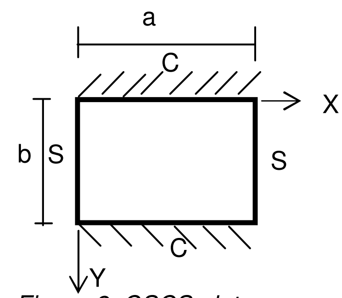

Figure 3: CSCS plate

THIN PLATE THEORY

The free harmonic vibration of a thin plate with constant thickness h is governed by the differential equation given by Jiu, et al (2007) as;

$$\nabla^4 W_{(x,y)} - \frac{\omega^2 \rho h}{D} W_{(x,y)} = 0 \qquad (1)$$

Where; ∇^4 is the biharmonic differential operator (i.e., $\nabla^4 = \nabla^2\nabla^2$)

$$\nabla^4 W_{(x,y)} = \frac{\partial^4}{\partial x^4} W_{(x,y)} + 2\frac{\partial^4}{\partial x^2 \partial y^2} W_{(x,y)} + \frac{\partial^4}{\partial y^4} W_{(x,y)} \qquad (2)$$

ω is the natural circular frequency of the vibrating plate given in rad/sec.

ρ is mass density of the plate.

h is thickness of the plate.

BOUNDARY CONDITIONS

For a thin rectangular flat plate with edge lengths a and b, there are eight boundary conditions for every case. Three cases are discussed below: (i) fully simply supported, SSSS. (ii) Fully clamped, CCCC. (iii) Two opposite edges clamped and the other edges simply supported, CSCS.

SSSS Plate: W (0, y) =0; W (a,y) = 0; W(x,0) = 0; W(x,b) = 0; $W''^{x}(0,y) = 0$;

$$W''^{x}(a,y) = 0; W''^{y}(x,0) = 0; W''^{y}(x,b) = 0; \qquad (3)$$

$$\text{Where } W''^{x} = \frac{\partial^2 W}{\partial x^2} \; and \; W''^{y} = \frac{\partial^2 W}{\partial y^2}$$

CCCC Plate: W (0, y) =0; W (a,y) = 0; W(x,0) = 0; W(x,b) = 0; $W'^{x}(0,y) = 0$;

$$W'^{x}(a,y) = 0; W'^{y}(x,0) = 0; W'^{y}(x,b) = 0; \qquad (4)$$

$$\text{Where } W'^{x} = \frac{\partial W}{\partial x} \; and \; W'^{y} = \frac{\partial W}{\partial y}$$

CSCS Plate: W (0, y) =0; W (a,y) = 0; W(x,0) = 0; W(x,b) = 0; $W''^{x}(0,y) = 0$;

$$W''^{x}(a,y) = 0;\; W'^{y}(x,0) = 0; W'^{y}(x,b) = 0. \qquad (5)$$

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ORDINARY FINITE DIFFERENCE COEFFICIENTS AND PATTERNS

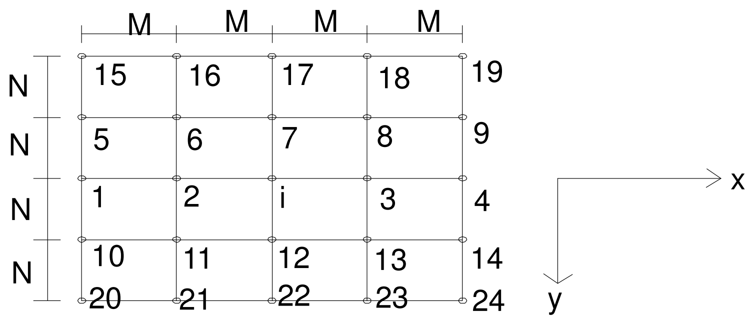

Figure 4. Diagram showing a discretized Rectangular Plate

Where P = M / N is the aspect ratio. M and N are the spans of each rectangular panel.

Point i(0,0) is taken as the origin. Using the Central Difference, the Ordinary Finite-difference coefficients for the differentials are given below as:

$$\frac{\partial W}{\partial x} = \frac{1}{2\mathrm{M}}\left[W_{(M,0)} - W_{(-M,0)}\right] = \frac{1}{2\mathrm{M}}[W_3 - W_2] \quad (6)$$

$$\frac{\partial W}{\partial y} = \frac{1}{2\mathrm{N}}\left[W_{(0,N)} - W_{(0,-N)}\right] = \frac{1}{2\mathrm{M}}[W_{12} - W_7] \quad (7)$$

$$\frac{\partial^2 W}{\partial x^2} = \frac{1}{\mathrm{M}^2}\left[W_{(M,0)} - 2W_{(0,0)} + W_{(-M,0)}\right] = \frac{1}{\mathrm{M}^2}[W_3 - 2W_i + W_2] \quad (8)$$

$$\frac{\partial^2 W}{\partial y^2} = \frac{1}{\mathrm{N}^2}\left[W_{(0,N)} - 2W_{(0,0)} + W_{(0,-N)}\right] = \frac{1}{\mathrm{N}^2}[W_{12} - 2W_i + W_7] \quad (9)$$

$$\frac{\partial^4 W}{\partial x^4} = {}^{1}/_{M^4}\left[W_{(2m,0)} - 4W_{(m,0)} + 6W_{(0,0)} - 4W_{(-m,0)} + W_{(-2m,0)}\right]$$

$$= {}^{1}/_{M^4}[W_4 - 4W_3 + 6W_i - 4W_2 + W_1] \quad (10)$$

$$\frac{\partial^4 W}{\partial y^4} = {}^{1}/_{N^4}\left[W_{(0,2N)} - 4W_{(0,N)} + 6W_{(0,0)} - 4W_{(0,-N)} + W_{(0,-2N)}\right.$$

$$= {}^{1}/_{N^4}[W_{22} - 4W_{12} + 6W_i - 4W_7 + W_{17}] \quad (11)$$

$$2\frac{\partial^4 W}{\partial x^2 \partial y^2} = \frac{1}{M^2 N^2}[2W_{13} - 4W_3 + 2W_8 - 4W_{12} + 8W_i - 4W_7 + 2W_{11} - 4W_2 + 2W_6]$$

$$\mathrm{N}^4\nabla^4 = \frac{1}{P^4}[W_4 - 4W_3 + 6W_i - 4W_2 + W_1] + [W_{22} - 4W_{12} + 6W_i - 4W_7 + W_{17}] +$$

$$\frac{1}{P^2}[2W_{13} - 4W_3 + 2W_8 - 4W_{12} + 8W_i - 4W_7 + 2W_{11} - 4W_2 + 2W_6] \quad (12)$$

The Patterns are given as;

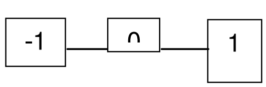

Figure 4: Pattern for $\frac{\partial W}{\partial x}$.

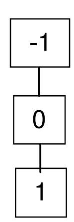

Figure 5: Pattern for $\frac{\partial W}{\partial y}$.

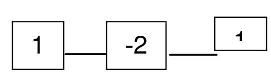

Figure 6: Pattern for $\frac{\partial^2 W}{\partial x^2}$.

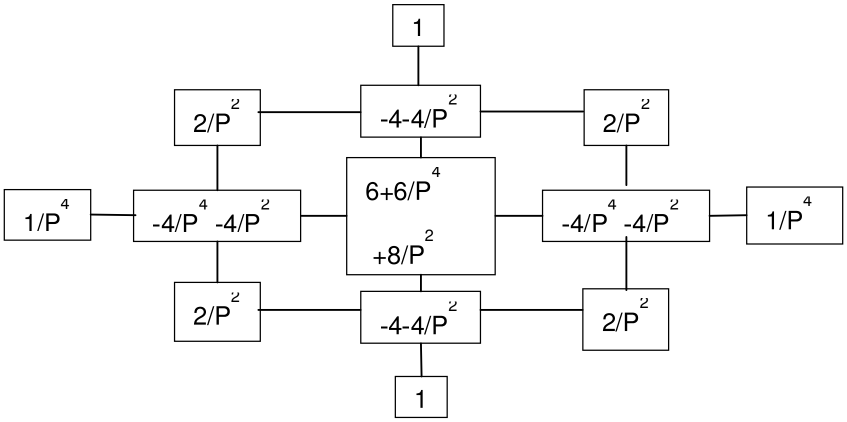

Figure 7. Pattern for∇^4

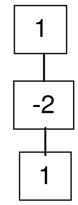

Figure 8. Pattern for $\frac{\partial^2 w}{\partial y^2}$.

The ordinary finite-difference coefficients and patterns derived were used to replace the derivatives in the governing equation and applied at each of the internal nodes noting the boundary conditions to generate the required eigenvalue equation.

EIGENVALUE EQUATION

Equation (1) yields an eigenvalue equation given as;

$$(A - \lambda B)x = 0 \qquad (13)$$

Where;

$x = [x_i]$ is a column matrix whose element x_i represent the amplitudes of the free vibration.

$A = [a_{ij}]$is a square matrix obtained from the ordinary finite–difference expression of the biharmonic operator ∇^4.

$B = [b_{ij}]$ is a diagonal matrix representing the constant $\frac{\rho h}{D}$.

$\lambda = \omega^2$, the circular frequency

Pre-multiplying eq. (2) by B^{-1}, we obtain;

$$(C - \lambda I)x = 0 \qquad (14)$$

This is the general eigenvalue equation for free-vibration of thin rectangular flat plates.

Where; $C = B^{-1}A \text{ and } I$ is an identity matrix.

For non – trivial solution of eq. (14), ie. $x \neq 0$, the determinants of the coefficients must varnish;

$$\therefore |C - \lambda I| = \begin{vmatrix} C_{11} - \lambda & C_{12} & C_{13} \; \dots & C_{1n} \\ C_{21} & C_{22} - \lambda & C_{23} \; \dots & C_{2n} \\ C_{31} & C_{32} & C_{33} - \lambda \; \dots & C_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ C_{n1} & C_{n2} & C_{n3} & C_{nn} - \lambda \end{vmatrix} = 0 \qquad (15)$$

The expansion of equation (15) yields a polynomial equation of n^{th}– order in λ, and this is called the characteristic equation. The roots $\lambda_i (i = 1,2,3, \dots, n)$ are the eigenvalues of the vibrating system from which the natural circular frequencies $\omega_i = \sqrt{\lambda}$ (i=1,2,3, …,n) are calculated (Szilard, 2004).

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The ordinary finite-difference coefficients and patterns derived were used to write a Visual Basic program which generated the required eigen value equation as well as the eigen values and eigenvectors for the SSSS, CCCC and CSCS plates when the grid sizes, n and the Aspect ratios a/b were entered.

RESULTS AND DISCUSSIONS

The fundamental natural frequency is given as;

$$\omega_1 = \frac{\lambda}{a^2}\sqrt{\frac{D}{\rho h}}\ rad/s$$, where h is the thickness of the plate.

Table 1. Values of λ for SSSS plate from the various methods

Aspect Ratio a/b	*Ventsel& Krauthammer (2001) 'V'*	*Chakraverty (2009) 'C'*	*Present Study. 'P'*	*% Difference 'P' and 'V'*
1	19.739	19.739	18.750	-5.275

Comparison of the present solution and two exact methods was shown in table 1. From the table, it is evident that the solution from Chakraverty (2009) and that of Ventsel& Krauthammer (2001) are the same. This is an exact solution.

The percentage difference between solution from the present study and that of Chakraverty (2009), is -5.275%. This difference is small and within the range of acceptance in statistics. Thus, the present study in this case gives a close approximation to the exact solution. However, the solution from the present study is a lower bound solution.

Table 2. Values of λ for CCCC plate from the various methods

Chakraverty (2009) 'C'	*Szilard (2004) 'S'*	*Present Study. 'P'*	*% Difference 'P' and 'C'*	*% Difference 'P' and 'S'*
35.988	28.80	28.80	-24.96	0

From table 2, it is evident that the solution from Szilard (2004) and that of the present study are the same. This is not a surprise since the two solutions are based on the ordinary finite difference method.

The percentage difference between solution from the present study and that of Chakraverty (2009), is -24.96%. This difference is large. This is because the ordinary finite difference method in this case does not give a better result beyond the grid size of 3. Thus in this case, it approximates slowly to the exact solution. However, the solution from the present study is a lower bound solution.

Table 3. Values of λ for CSCS plate from the various methods

Chakraverty (2009) 'C'	*Gorman (1982) 'G'*	*Present Study. 'P'*	*% Difference 'P' and 'C'*	*% Difference 'G' & 'C'*
28.95	28.93	24.29	-19.18	-0.069

Table 3 shows the comparison of the present result with that of Chakraverty (2009) and Gorman (1982). From the table, the percentage difference between solution from the present study and that of Chakraverty (2009), is -19.18%. This difference is large. This is because the ordinary finite difference method (OFDM) in this case does not give a better result beyond the grid size of 3. Thus in this case, it approximates slowly to the exact solution. However, the solution from the present study is a lower bound solution.

Comparison between the solutions from Chakraverty (2009) and Gorman (1982) shows that the two results are approximately equal. The exact solution is somewhat around these two solutions.

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