

OPTIMIZATION OF COMPRESSIVE STRENGTH OF FLY ASH BLENDED CEMENT CONCRETE USING SCHEFFE'S SIMPLEX THEORY

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ABSTRACT

High cost of cement as a major component of concrete has contributed to continuous rise in the cost of concrete works in Nigeria and other developing nations. Blended cement has become quite popular in developed countries due to its durability and high benefits / cost ratio. Fly ash as a residue resulting from combustion of pulverized coal or lignite occurs in large quantities in some parts of Nigeria. Generally, mix proportions of the various components determine the compressive strength and other properties of concrete. In this study, a mathematical model was developed for optimizing the compressive strength of fly ash blended cement concrete based on Scheffe's Simplex Polynomial theory. A total of ninety (90) cubes were cast, consisting of three cubes per mix ratio and for a total of thirty (30) mix ratios. The first fifteen (15) were used to determine the coefficients of the model, while the other fifteen were used to validate the model. The five component second degree (5, 2) mathematical model compared favourably with the experimental data and the predictions from the model were tested with the statistical Fischer test and found to be adequate at 95% confidence level. The optimum compressive strength of the blended concrete at twenty-eight (28) days was found to be 43.152 N/mm². This strength corresponded to a mix ratio of 0.549:0.935:0.065:1.760:3.52 for water: cement: fly ash: sand: granites respectively. The model derived in this study can be used to predict mix ratios for any desired strength of fly ash blended cement concrete within the factor space of the simplex used in the study and vice versa.

Keywords: Blended Cement, Compressive Strength, Concrete, Fly Ash, Mathematical model, Optimization.

INTRODUCTION

One of the basic needs of man is housing. In many developing countries like Nigeria, there is a perpetual problem of accommodation and inadequate housing. A recent research showed that about seven million Nigerians have no accommodation (Uwe, 2010). It is important to note that majority of housing units in Nigeria are constructed using concrete, which has ordinary Portland cement as a basic constituent. In fact, twice as much concrete is used in construction in Nigeria than the total of all other building materials.

According to Neville and Brook (1990), concrete is a product of water, cement and aggregate, and when sufficiently hardened, is used in carrying various loads. However, in Nigeria due to the rapid rise in the cost of ordinary Portland cement (which is an important ingredient in concrete) there is need to develop cheap and replaceable substitute for cement. Consequently, many researches are being carried out on cheap and replaceable or complimentary substitutes for ordinary Portland cement. These researches are also aimed at putting into effective use industrial waste products.

The properties of concrete are controlled by the relative quantities of cement, aggregates and water mixed together both in plastic and hardened states. Also these properties can be improved by the addition of either a chemical admixture or supplementary cementitious

material, which will make the number of components of concrete five (that is in addition to water, cement, coarse aggregate and fine aggregate). For this research work the components of concrete are cement, fly ash, fine aggregate, coarse aggregate and water.

The aim of this study is to develop mathematical model for the optimization of Compressive Strength of fly ash blended cement concrete based on Scheffe's (5, 2) polynomial equation. Different percentages of fly ash were used for partial replacement of cement. This involved testing concrete from the different mix ratios where cement is partially replaced with fly ash and developing a mathematical model that can be used to predict the compressive strength of concrete given any mix ratio or predict mix ratios given a particular Compressive Strength of concrete. As the number of components increased, cost per m³ increased, making optimization of concrete mixtures necessary so as to obtain concrete with required and suitable properties at minimum cost.

SCHEFFE'S (5, 2) SIMPLEX DESIGN

Response equation of Scheffe's (5, 2) simplex design was given by (Obam, 2006) as:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (1)$$

Where β_i and X_i are the coefficients of response equation and pseudo components of the mix respectively.

The coefficients in terms of pseudo components, X_i and laboratory responses of the first fifteen ratios gave the following relations as the regression model of equation Scheffe's (5, 2) simplex design (Ezeh, et.al, 2010).

$$Y = n_1 X_1 (2X_1 - 1) + n_2 X_2 (2X_2 - 1) + n_3 X_3 (2X_3 - 1) + n_4 X_4 (2X_4 - 1) + n_5 X_5 (2X_5 - 1) + 4n_{12} X_1 X_2 + 4n_{13} X_1 X_3 + 4n_{14} X_1 X_4 + 4n_{15} X_1 X_5 + 4n_{23} X_2 X_3 + 4n_{24} X_2 X_4 + 4n_{25} X_2 X_5 + 4n_{34} X_3 X_4 + 4n_{35} X_3 X_5 \quad (2)$$

MIX RATIOS

Five mixed ratios (real and pseudo) that defined the vertices pentahedron simplex lattice used in this study are shown in Table 1.

Table 1. First Five Mix Ratios (Actual and Pseudo) Obtained From Scheffe's (5, 2) factor space

| Points | Actual Mix ratios | | | | | Pseudo Mix ratios | | | | |
|----------------|-------------------|-----------------|------------------|---------------|------------------|-------------------|-----------------|------------------|---------------|------------------|
| | Water S_1 | Cement S_2 | Fly Ash S_3 | Sand S_4 | Granite S_5 | Water X_1 | Cement X_2 | Fly Ash X_3 | Sand X_4 | Granite X_5 |
| N ₁ | 0.57 | 0.95 | 0.05 | 2.0 | 4.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| N ₂ | 0.50 | 0.90 | 0.10 | 1.2 | 2.4 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| N ₃ | 0.55 | 0.85 | 0.15 | 1.5 | 2.0 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |
| N ₄ | 0.60 | 0.8 | 0.20 | 2.1 | 4.2 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| N ₅ | 0.60 | 0.75 | 0.25 | 1.2 | 4.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |

According to Osadebe and Ibearugbulem (2009), the actual mix ratios relate with pseudo mix ratios in the mathematical form:

$$\{S\} = [A]\{X\} \tag{3}$$

Where S , A and X , represent the actual mix ratio, coefficient of relation matrix, and pseudo mix ratio respectively. Matrix A can be taken to be the transpose of the first five actual mix ratios shown in Table 1 and this resulted to:

$$[A] = \begin{pmatrix} 0.57 & 0.5 & 0.55 & 0.60 & 0.60 \\ 0.95 & 0.9 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.1 & 0.15 & 0.20 & 0.25 \\ 2.00 & 1.2 & 1.50 & 2.10 & 1.20 \\ 4.00 & 2.4 & 2.00 & 4.20 & 4.00 \end{pmatrix} \tag{4}$$

The five actual and pseudo mix ratios in table 1 correspond to points of observations, N_1, N_2, N_3, N_4, N_5 located at the five vertices of the pentahedron. For a (5, 2) simplex design, ten other observations are needed to add up to the first five to get a total of fifteen observations. This was used to formulate the model. The remaining ten points were located at the mid points of the lines joining the five vertices. On substitution of these ten pseudo mix ratios, one after the other into equation 3, the real mix ratios corresponding to the pseudo ones were obtained. Their values are shown in Table 2.

Table 2. Remaining Ten Mix Ratios (Actual and Pseudo) for formulation of the Model

| Points | Actual Mix ratios | | | | | Pseudo Mix ratios | | | | |
|----------|-------------------|-----------------|------------------|---------------|------------------|-------------------|-----------------|------------------|---------------|------------------|
| | Water S_1 | Cement S_2 | Fly Ash S_3 | sand S_4 | Granite S_5 | Water X_1 | Cement X_2 | Fly Ash X_3 | sand X_4 | Granite X_5 |
| N_{12} | 0.535 | 0.925 | 0.075 | 1.6 | 3.2 | 0.5 | 0.5 | 0.0 | 0.0 | 0.0 |
| N_{13} | 0.56 | 0.90 | 0.10 | 1.75 | 3.0 | 0.5 | 0.0 | 0.5 | 0.0 | 0.0 |
| N_{14} | 0.585 | 0.875 | 0.125 | 2.05 | 4.1 | 0.5 | 0.0 | 0.0 | 0.5 | 0.0 |
| N_{15} | 0.585 | 0.85 | 0.15 | 1.6 | 4.0 | 0.5 | 0.0 | 0.0 | 0.0 | 0.5 |
| N_{23} | 0.525 | 0.875 | 0.125 | 1.35 | 2.2 | 0.0 | 0.5 | 0.5 | 0.0 | 0.0 |
| N_{24} | 0.55 | 0.85 | 0.15 | 1.65 | 3.3 | 0.0 | 0.5 | 0.0 | 0.5 | 0.0 |
| N_{25} | 0.55 | 0.825 | 0.175 | 1.2 | 3.2 | 0.0 | 0.5 | 0.0 | 0.0 | 0.5 |
| N_{34} | 0.575 | 0.825 | 0.175 | 1.8 | 3.1 | 0.0 | 0.0 | 0.5 | 0.5 | 0.0 |
| N_{35} | 0.575 | 0.80 | 0.20 | 1.35 | 3.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.5 |
| N_{45} | 0.60 | 0.775 | 0.225 | 1.65 | 4.1 | 0.0 | 0.0 | 0.0 | 0.5 | 0.5 |

In order to validate the model, extra fifteen points ($C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}$, and C_{15}) of observations were used. These observations served as control mix

ratios of the concrete mixes in this research. The mix ratios (actual and Pseudo) for the work are shown in table 3, while that of the thirty mix ratios (comprising 15 mix ratios for the trial mixes and 15 for control mixes) are shown in Table 4.

Table 3. Actual and pseudo components of fifteen control points of observation

| <i>S/N</i> | <i>S₁</i> | <i>S₂</i> | <i>S₃</i> | <i>S₄</i> | <i>S₅</i> | <i>X₁</i> | <i>X₂</i> | <i>X₃</i> | <i>X₄</i> | <i>X₅</i> |
|-----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| <i>C₁</i> | 0.54 | 0.9 | 0.1 | 1.567 | 2.80 | 0.333 | 0.333 | 0.333 | 0 | 0 |
| <i>C₂</i> | 0.573 | 0.867 | 0.133 | 1.866 | 3.40 | 0.333 | 0 | 0.333 | 0.333 | 0 |
| <i>C₃</i> | 0.590 | 0.833 | 0.167 | 1.766 | 5.066 | 0.333 | 0 | 0 | 0.333 | 0.333 |
| <i>C₄</i> | 0.555 | 0.875 | 0.125 | 1.7 | 3.15 | 0.25 | 0.25 | 0.25 | 0.25 | 0 |
| <i>C₅</i> | 0.580 | 0.8375 | 0.1635 | 1.7 | 3.55 | 0.25 | 0 | 0.25 | 0.25 | 0.25 |
| <i>C₆</i> | 0.555 | 0.8625 | 0.1375 | 1.475 | 3.1 | 0.25 | 0.25 | 0.25 | 0 | 0.25 |
| <i>C₇</i> | 0.546 | 0.925 | 0.0875 | 1.675 | 3.1 | 0.5 | 0.25 | 0.25 | 0 | 0 |
| <i>C₈</i> | 0.580 | 0.825 | 0.175 | 1.475 | 3.5 | 0.25 | 0 | 0.25 | 0 | 0.5 |
| <i>C₉</i> | 0.585 | 0.89 | 0.11 | 1.76 | 3.32 | 0.4 | 0.2 | 0.2 | 0.2 | 0 |
| <i>C₁₀</i> | 0.569 | 0.85 | 0.15 | 1.6 | 3.32 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| <i>C₁₁</i> | 0.571 | 0.855 | 0.145 | 1.68 | 3.48 | 0.3 | 0.1 | 0.2 | 0.2 | 0.2 |
| <i>C₁₂</i> | 0.567 | 0.835 | 0.165 | 1.61 | 3.34 | 0.1 | 0.2 | 0.2 | 0.3 | 0.2 |
| <i>C₁₃</i> | 0.562 | 0.8675 | 0.1325 | 1.555 | 3.26 | 0.35 | 0.15 | 0.25 | 0 | 0.25 |
| <i>C₁₄</i> | 0.562 | 0.855 | 0.145 | 1.625 | 3.42 | 0.25 | 0.2 | 0.15 | 0.2 | 0.2 |
| <i>C₁₅</i> | 0.582 | 0.8573 | 0.1425 | 1.74 | 3.96 | 0.45 | 0.05 | 0 | 0.2 | 0.3 |

Table 4. Mix ratios for thirty observations (Actual and Pseudo) obtained from Scheffe's (5, 2) factor space

| <i>Points</i> | <i>Actual Mix ratios</i> | | | | | <i>Pseudo Mix ratios</i> | | | | |
|----------------------|--------------------------------|---------------------------------|---------------------------------|-------------------------------|----------------------------------|--------------------------------|---------------------------------|---------------------------------|-------------------------------|----------------------------------|
| | <i>Water S₁</i> | <i>Cement S₂</i> | <i>FlyAsh S₃</i> | <i>sand S₄</i> | <i>Granite S₅</i> | <i>Water X₁</i> | <i>Cement X₂</i> | <i>FlyAsh X₃</i> | <i>sand X₄</i> | <i>Granite X₅</i> |
| <i>N₁</i> | 0.57 | 0.95 | 0.05 | 2.0 | 4.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| <i>N₂</i> | 0.50 | 0.90 | 0.10 | 1.2 | 2.4 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| <i>N₃</i> | 0.55 | 0.85 | 0.15 | 1.5 | 2.0 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |
| <i>N₄</i> | 0.60 | 0.8 | 0.20 | 2.1 | 4.2 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| <i>N₅</i> | 0.60 | 0.75 | 0.25 | 1.2 | 4.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |

| | | | | | | | | | | |
|-----------------|-------|--------|--------|-------|-------|-------|-------|-------|-------|-------|
| N ₁₂ | 0.535 | 0.925 | 0.075 | 1.6 | 3.2 | 0.5 | 0.5 | 0.0 | 0.0 | 0.0 |
| N ₁₃ | 0.56 | 0.90 | 0.10 | 1.75 | 3.0 | 0.5 | 0.0 | 0.5 | 0.0 | 0.0 |
| N ₁₄ | 0.585 | 0.875 | 0.125 | 2.05 | 4.1 | 0.5 | 0.0 | 0.0 | 0.5 | 0.0 |
| N ₁₅ | 0.585 | 0.85 | 0.15 | 1.6 | 4.0 | 0.5 | 0.0 | 0.0 | 0.0 | 0.5 |
| N ₂₃ | 0.525 | 0.875 | 0.125 | 1.35 | 2.2 | 0.0 | 0.5 | 0.5 | 0.0 | 0.0 |
| N ₂₄ | 0.55 | 0.85 | 0.15 | 1.65 | 3.3 | 0.0 | 0.5 | 0.0 | 0.5 | 0.0 |
| N ₂₅ | 0.55 | 0.825 | 0.175 | 1.2 | 3.2 | 0.0 | 0.5 | 0.0 | 0.0 | 0.5 |
| N ₃₄ | 0.575 | 0.825 | 0.175 | 1.8 | 3.1 | 0.0 | 0.0 | 0.5 | 0.5 | 0.0 |
| N ₃₅ | 0.575 | 0.80 | 0.20 | 1.35 | 3.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.5 |
| N ₄₅ | 0.60 | 0.775 | 0.225 | 1.65 | 4.1 | 0.0 | 0.0 | 0.0 | 0.5 | 0.5 |
| <i>CONTROL</i> | | | | | | | | | | |
| C ₁ | 0.54 | 0.9 | 0.1 | 1.567 | 2.80 | 0.333 | 0.333 | 0.333 | 0 | 0 |
| C ₂ | 0.573 | 0.867 | 0.133 | 1.866 | 3.40 | 0.333 | 0 | 0.333 | 0.333 | 0 |
| C ₃ | 0.590 | 0.833 | 0.167 | 1.766 | 5.066 | 0.333 | 0 | 0 | 0.333 | 0.333 |
| C ₄ | 0.555 | 0.875 | 0.125 | 1.7 | 3.15 | 0.25 | 0.25 | 0.25 | 0.25 | 0 |
| C ₅ | 0.580 | 0.8375 | 0.1635 | 1.7 | 3.55 | 0.25 | 0 | 0.25 | 0.25 | 0.25 |
| C ₆ | 0.555 | 0.8625 | 0.1375 | 1.475 | 3.1 | 0.25 | 0.25 | 0.25 | 0 | 0.25 |
| C ₇ | 0.546 | 0.925 | 0.0875 | 1.675 | 3.1 | 0.5 | 0.25 | 0.25 | 0 | 0 |
| C ₈ | 0.580 | 0.825 | 0.175 | 1.475 | 3.5 | 0.25 | 0 | 0.25 | 0 | 0.5 |
| C ₉ | 0.585 | 0.89 | 0.11 | 1.76 | 3.32 | 0.4 | 0.2 | 0.2 | 0.2 | 0 |
| C ₁₀ | 0.569 | 0.85 | 0.15 | 1.6 | 3.32 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| C ₁₁ | 0.571 | 0.855 | 0.145 | 1.68 | 3.48 | 0.3 | 0.1 | 0.2 | 0.2 | 0.2 |
| C ₁₂ | 0.567 | 0.835 | 0.165 | 1.61 | 3.34 | 0.1 | 0.2 | 0.2 | 0.3 | 0.2 |
| C ₁₃ | 0.562 | 0.8675 | 0.1325 | 1.555 | 3.26 | 0.35 | 0.15 | 0.25 | 0 | 0.25 |
| C ₁₄ | 0.562 | 0.855 | 0.145 | 1.625 | 3.42 | 0.25 | 0.2 | 0.15 | 0.2 | 0.2 |
| C ₁₅ | 0.582 | 0.8573 | 0.1425 | 1.74 | 3.96 | 0.45 | 0.05 | 0 | 0.2 | 0.3 |

MATERIALS

The materials used for the laboratory test included:

- I. Water that is good for drinking obtained from a well-treated borehole at the premises of Imo State Ministry of Works, Owerri, Nigeria. The water was clean, fresh and free from dirt, unwanted chemicals or rubbish that may affect the desired quality of concrete.
- II. Dangote cement, a brand of ordinary Portland cement that conforms to BS 12(1978).
- III. River sand obtained from Otamiri River was the fine aggregate. The river sand was sharp and free from clay, debris and other deleterious materials. The grading of the sand was carried out to BS 812:103 (BS 812: Part 1, 1975). The sand belongs to grading zone C (Neville, A.M., 1996).
- IV. The coarse aggregate used for this research work were granite chippings quarried from crushed rock industries quarry, Ishiagu, along Enugu-Port Harcourt express way, Ebonyi state, Nigeria. The granite has a maximum size of 20mm. They were washed and sun-dried for seven days in the laboratory to ensure that they were free from excessive dust, and organic matter.
- V. Fly ash, otherwise known as pulverized fuel ash (PFA) is a pozzolanic material. Fly ash used as a partial replacement of cement in various mix proportion was obtained from the thermal coal station at Oji River, Enugu state, Nigeria. It was grinded and sieved with 212 μ m sieve to obtain finer particles capable of reacting with cement, and mixing with fine aggregate, coarse aggregate and water to form fly ash blended cement concrete.

The mix ratios used for the simplex design points were obtained using four-dimensional simplex lattice factor space for five-component two-degree mixture.

COMPRESSIVE STRENGTH TEST

Batching of the ingredients was done by mass. Cement/ fly ash was thoroughly mixed together with a mixture of sand and granite. The entire component was cast in concrete mould of size 150 x 150x 150 mm. The concrete cubes were cured in a curing tank for 28 days and were crushed using universal testing machine. Compressive strength of the cubes was calculated using equation 4:

$$\text{Compressive strength} = \frac{\text{compressive load of cube at failure (N)}}{\text{cross sectional area of mould (mm}^2\text{)}} \quad (4)$$

Compressive strengths of the concrete cubes from the laboratory as obtained from the thirty points of observations are shown in Table 5.

Table 5. Compressive strength in N/mm² of 28 day old concrete cubes

| <i>Points</i> | <i>Replicate 1</i> | <i>Replicate 2</i> | <i>Replicate 3</i> | <i>Mean Values</i> | <i>Model Values</i> |
|---------------|--------------------|--------------------|--------------------|--------------------|---------------------|
| N_1 | 42.66 | 42.66 | 42.22 | 42.51 | 42.51 |
| N_2 | 34.22 | 39.55 | 40.88 | 38.21 | 38.21 |
| N_3 | 31.55 | 34.46 | 34.22 | 33.41 | 33.41 |
| N_4 | 41.33 | 42.66 | 38.22 | 40.73 | 40.73 |

| | | | | | |
|----------|-------|-------|-------|-------|-------|
| N_5 | 21.33 | 24.44 | 20.00 | 21.92 | 21.92 |
| N_{12} | 41.33 | 39.11 | 47.55 | 42.66 | 42.66 |
| N_{13} | 40.88 | 40.00 | 40.77 | 40.55 | 40.55 |
| N_{14} | 35.11 | 35.55 | 32.00 | 34.22 | 34.22 |
| N_{15} | 31.11 | 28.44 | 28.88 | 29.47 | 29.47 |
| N_{23} | 30.22 | 32.88 | 31.11 | 31.40 | 31.40 |
| N_{24} | 33.77 | 34.22 | 33.77 | 33.92 | 33.92 |
| N_{25} | 33.33 | 31.11 | 35.55 | 33.23 | 33.33 |
| N_{34} | 26.66 | 29.33 | 29.77 | 28.58 | 28.58 |
| N_{35} | 24.88 | 29.33 | 27.11 | 27.10 | 27.10 |
| N_{45} | 17.77 | 18.66 | 18.66 | 18.36 | 18.36 |
| C_1 | 38.11 | 42.66 | 43.55 | 41.44 | 38.94 |
| C_2 | 33.00 | 33.88 | 32.44 | 33.12 | 32.87 |
| C_3 | 33.55 | 20.11 | 26.22 | 23.29 | 24.69 |
| C_4 | 30.88 | 34.66 | 39.55 | 35.03 | 33.48 |
| C_5 | 24.55 | 24.11 | 24.55 | 24.40 | 27.25 |
| C_{12} | 33.33 | 32.88 | 33.33 | 33.18 | 34.12 |
| C_{13} | 40.40 | 40.00 | 40.77 | 40.39 | 40.50 |
| C_{14} | 28.44 | 29.33 | 28.88 | 28.88 | 28.93 |
| C_{15} | 39.11 | 34.67 | 38.22 | 37.33 | 35.72 |
| C_{23} | 29.33 | 30.11 | 29.22 | 29.55 | 29.92 |
| C_{24} | 29.11 | 28.66 | 30.44 | 29.40 | 30.19 |
| C_{25} | 25.00 | 25.66 | 25.88 | 25.51 | 27.71 |
| C_{34} | 32.00 | 32.44 | 33.77 | 32.77 | 38.56 |
| C_{35} | 27.11 | 28.11 | 29.11 | 28.11 | 30.57 |
| C_{45} | 28.00 | 28.88 | 28.88 | 28.59 | 28.69 |

MODEL FOR PREDICTING THE COMPRESSIVE STRENGTH OF THE CONCRETE

This model is obtained by substituting compressive strength in table 5 of concrete cubes from the first fifteen points of observations ($N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14},$ and N_{15}) into equation 2 to obtain:

$$Y = 42.51X_1 (2X_1 - 1) + 38.21(2X_2 - 1) X_2 + 33.41X_3 (2X_3 - 1) + 40.73 X_4 (2X_4 - 1) + 21.92$$

$$(2X_5 - 1)X_5 + 170.69X_1X_2 + 158.2X_1X_3 + 136.88X_1X_4 + 117.88X_1X_5 + 125.6X_2X_3 + 135.68X_2X_4 + 133.32X_2X_5 + 114.32X_3X_4 + 108.4X_3X_5 + 73.44 X_4 X_5 \quad (6)$$

Equation (6) is the mathematical model for the optimization of compressive strength of Fly Ash Blended Cement concrete based on Scheffe's (5, 2) factor space.

TEST FOR ADEQUACY OF THE MODEL

The test for adequacy of the model was done using Fischer test at 95% confidence level on the compressive strength at the control points (that is, C₁, C₂, C₃, C₄, C₅, C₆, C₇, C₈, C₉, C₁₀, C₁₁, C₁₂, C₁₃, C₁₄, and C₁₅). In this test, two hypotheses were set as follows:

Null Hypothesis

There is no significant difference between the laboratory concrete cube strength and model predicted strength results.

Alternative Hypothesis

There is a significant difference between the laboratory concrete cube strength and model predicted strength results.

The test was carried out as shown in Table 6.

Table 6. Fischer-statistical test computations for the model

| Control points | y_e | y_m | $y_e - \bar{y}_e$ | $y_m - \bar{y}_m$ | $(y_e - \bar{y}_e)^2$ | $(y_m - \bar{y}_m)^2$ |
|-----------------|-----------------------|------------------------|-------------------|-------------------|--|--|
| C ₁ | 41.44 | 38.94 | 10.13 | 7.07 | 102.62 | 49.98 |
| C ₂ | 33.11 | 32.17 | 1.5 | 1.00 | 3.24 | 10.50 |
| C ₃ | 23.29 | 24.69 | -8.02 | -7.18 | 64.32 | 51.55 |
| C ₄ | 33.69 | 33.48 | 2.38 | 1.61 | 5.66 | 2.59 |
| C ₅ | 24.40 | 27.25 | -6.91 | -4.62 | 47.75 | 21.34 |
| C ₆ | 83.14 | 34.12 | 1.86 | 2.25 | 3.46 | 5.06 |
| C ₇ | 40.37 | 40.50 | 9.08 | 8.53 | 82.45 | 74.48 |
| C ₈ | 25.88 | 28.93 | -2.43 | -2.94 | 5.90 | 8.64 |
| C ₉ | 37.33 | 35.72 | 6.02 | 3.85 | 36.24 | 14.82 |
| C ₁₀ | 29.55 | 29.92 | -1.39 | -1.95 | 1.93 | 3.80 |
| C ₁₁ | 29.40 | 30.19 | -1.91 | -1.68 | 3.65 | 2.82 |
| C ₁₂ | 25.51 | 27.71 | -5.8 | -4.16 | 33.64 | 17.31 |
| C ₁₃ | 32.74 | 34.56 | 1.43 | 2.69 | 2.04 | 7.24 |
| C ₁₄ | 28.11 | 30.57 | -3.20 | -1.3 | 10.24 | 1.69 |
| C ₁₅ | 28.59 | 28.69 | -2.72 | -3.18 | 7.40 | 10.11 |
| Sum | $\sum y_e$ = 469.6 | $\sum y_m$ = 478.05 | | | $\sum (y_e - \bar{y}_e)^2$ = 410.54 | $\sum (y_m - \bar{y}_m)^2$ = 281.98 |
| Mean | $\bar{y}_e = 31.31$ | $\bar{y}_m = 31.87$ | | | | |

Note: y_e is the experimental compressive strength, while y_m is the model compressive strength

$$S_e^2 = \frac{\sum (y_e - \bar{y}_e)^2}{N-1} = \frac{410.39}{14} = 29.32$$

$$S_m^2 = \frac{\sum (y_m - \bar{y}_m)^2}{N-1} = \frac{281.98}{14} = 20.14$$

$$F_{\text{calculated}} = \frac{S_1^2}{S_2^2}$$

Where S_1^2 is the greater of S_e^2 and S_m^2 , while S_2^2 is the smaller of the two.

Here, $S_1^2 = S_e^2 = 29.32$ and $S_2^2 = S_m^2 = 20.14$

$$F_{\text{calculated}} = \frac{29.32}{20.14} = 1.456$$

The model is acceptable at 95% confidence level if:

$$\frac{1}{F_{\alpha(V_1, V_2)}} < \frac{S_1^2}{S_2^2} < F_{\alpha(V_1, V_2)}$$

Where, Significant level, $\alpha = 1 - 0.95 = 0.05$; Degree of freedom, $V = N - 1 = 15 - 1 = 14$

From standard F-statistic table, $F_{\alpha(V_1, V_2)} = 2.443$ and, $\frac{1}{F_{\alpha(V_1, V_2)}} = \frac{1}{2.443} = 0.4093$

Hence the condition: $\frac{1}{F_{\alpha(V_1, V_2)}} < \frac{S_1^2}{S_2^2} < F_{\alpha(V_1, V_2)}$ which is $0.4093 < 1.456 < 2.443$, is satisfied.

Therefore, the null hypothesis that “there is no significant difference between the experimental and the model expected result” is accepted. This implies that the model equation is adequate.

CONCLUSION

Using Scheffe's (5, 2) polynomial equation, mix design mathematical model for a five component fly ash blended cement concrete was developed. This model could predict the compressive strength of fly ash blended concrete when the mix ratios are known and vice versa. The predictions from the model were tested at 95% accuracy level using statistical Fischer test and found to be adequate. The maximum strength predicted by this model was 43.152 N/mm^2 derived from a mix ratio of 0.549:0.935:0.065:1.760:3.52 for water: cement: fly ash: sand: granite respectively.

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