ISSN-L: 2223-9553, ISSN: 2223-9944
Vol. 4 No. 4 July 2013

FUZZY STABILIZATION OF A COUPLED LORENZ-ROSSLER CHAOTIC SYSTEM

Edwin A. Umoh

Department of Electrical Engineering Technology, Federal Polytechnic,
Kaura Namoda, NIGERIA.

eddyumoh@gmail.com

ABSTRACT

This paper reports the stabilization of a novel system realized by additively coupling the Lorenz and Rossler chaotic systems. The structural dynamics of the novel system is distinctively different and topologically nonequivalent to the of either the Lorenz or Rossler systems. A Lyapunov function-based fuzzy controller was designed and employed to drive the trajectories of the system to some equilibrium point at the origin, in the sense of Lyapunov. The various scenarios observed by manipulation of the system's equations during numerical simulations using MATLAB software are discussed. Overall, the results show that the novel system is stabilizable at some equilibrium points. However, the parameters still need some fine tuning to produce an elegant relationship that can evolves into a strange attractor.

Keywords: Fuzzy control, Lorenz system, Lyapunov stability theorem, Rossler system

INTRODUCTION

Since the discovery of chaotic dynamics in weather systems by Edward Lorenz in 1963 (Lorenz, 1963), and the subsequent proofs of the present of chaos in several natural and man-made systems, enormous intellectual and financial resources have been channeled towards unraveling the usefulness or otherwise of chaos for engineering and other scientific applications. These adventures led to the discovering of the Rossler, Chua, Matsumoto-Chua-Kobayashi (MCK), Rabinovich, Sprott family of systems amongst a large body of others which formed the earliest attractors coined and extensively research upon. Chaos is a phenomenon of deterministic dynamic systems which are extremely sensitive to perturbation of their initial conditions and whose long-term evolution is difficult to predict. The ubiquity and usefulness of chaos in modeling real systems has transformed the search for newer strange attractors into a fascinating subject that cuts across disciplines. In this connection, many new attractors have been found accidentally while the discovery of others have followed a systematic order involving rigorous mathematical proofs and characterization of their geometric and statistical properties (Lu et al., 2002). And strikingly, every newly evolved chaotic systems has been received with open hand by the scientific community because they could be relevant presently or in the future, to engineering and non-engineering applications in secure communications (Guan et al., 2002), medicine (Kumar and Hedge, 2012), economics and financial system modelling (Guegan, 2009), psychology (Robertson and Combs, 1995) amongst others. Although many new attractors have topologically nonequivalent structures to the original canonical systems such as the Lorenz, Chen or Rossler from which they were coined, nonetheless, their structural evolutions consist essentially in directly manipulation of the canonical system equations (Pelivan and Uyaroglu, 2010; Lu et al., 2004). The Lorenz equation is given by

$$\dot{x_1} = -\sigma x_1 + \sigma x_2$$

$$\dot{x_2} = \rho x_1 - x_2 - x_1 x_3$$

$$\dot{x}_3 = x_1x_2 - \beta x_3 \quad (1)$$

Where x_1, x_2 and x_3 are states of the system. For typical parameters of $\sigma = 10$, $\rho = 28, \beta = 8/3$, the famous butterfly attractor evolves. On the other hand, the Rossler system is represented by the following equations

$$\dot{x_1} = -x_2 - x_1$$

$$\dot{x_2} = x_1 + ax_2$$

$$\dot{x_3} = b + x_3(x_1 - c) \quad (2)$$

Where x_1, x_2 and x_3 are states of the system. For typical parameters of $= b = 0.2$ *and* $c = 5.7$, a strange attractor evolves.

THE LORENZ-ROSSLER CHAOTIC SYSTEM

The dynamics of the system was first reported by Alsafasfeh and Al-Arni (2011). The system was coined by adding the Lorenz and Rossler systems directly and interpolating some state variables to match the form given by Cuomo et al. (1993). The additively-coupled Lorenz-Rossler system is a three-dimensional autonomous system which has ten terms on the right hand side of the governing equations with three quadratic nonlinearities necessary for folding trajectories, five variational parameters and is described by the equation (3).

$$\dot{x_1} = \sigma(x_2 - x_1) - x_2 - x_1$$

$$\dot{x_2} = rx_1 - x_2 - 20x_1x_3 + x_1 + ax_2$$

$$\dot{x}_3 = 5x_1x_2 - \beta x_3 + b + x_1(x_3 - c) \quad (3)$$

Where $[x_1, x_2, x_3]^T \in \Re^3$ is the state variables of the systems, a, b and c are parameters. It is easy to verify that the system in (3) is globally, uniformly and asymptotically stable about its zero equilibrium if $a > 0, b > 0, c > 0$. the rate of volume contraction is given by the Lie derivative,

$$\frac{1}{V}\frac{dv}{dt} = \sum_i \frac{\partial \dot{\varphi}_i}{\partial \varphi_i}, i = 1,2,3;\ \varphi_1 = x, \varphi_2 = y, \varphi_3 = z \quad (4)$$

For the additively-coupled Lorenz-Rossler system, at the equilibrium point (0, 0, 0), we obtain

$$\frac{1}{V}\frac{dv}{dt} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\sigma - 1 + a - \beta = \gamma \quad (5)$$

For $\sigma = 20, \beta = 8.5, \gamma = -29.5 + a$. For $a < 29.5$, γ is negative and the system is dissipative. For case 1, with parameter values of $\sigma = 20, r = 20, a = 9, \beta = 8.5$, $b = 0, c = 8$, the system evolves the following attractors in Figure 1(a)- (c), and in case 2, when the constant associated with the nonlinearity $5x_1x_2$ is changed to $20x_1x_2$ with same parameter values as in case 1, the attractor evolved into a dense orbit shown in Figure 1(d)-(f). This mild change has significant influence of the stabilizability of the system.

ISSN-L: 2223-9553, ISSN: 2223-9944
Vol. 4 No. 4 July 2013

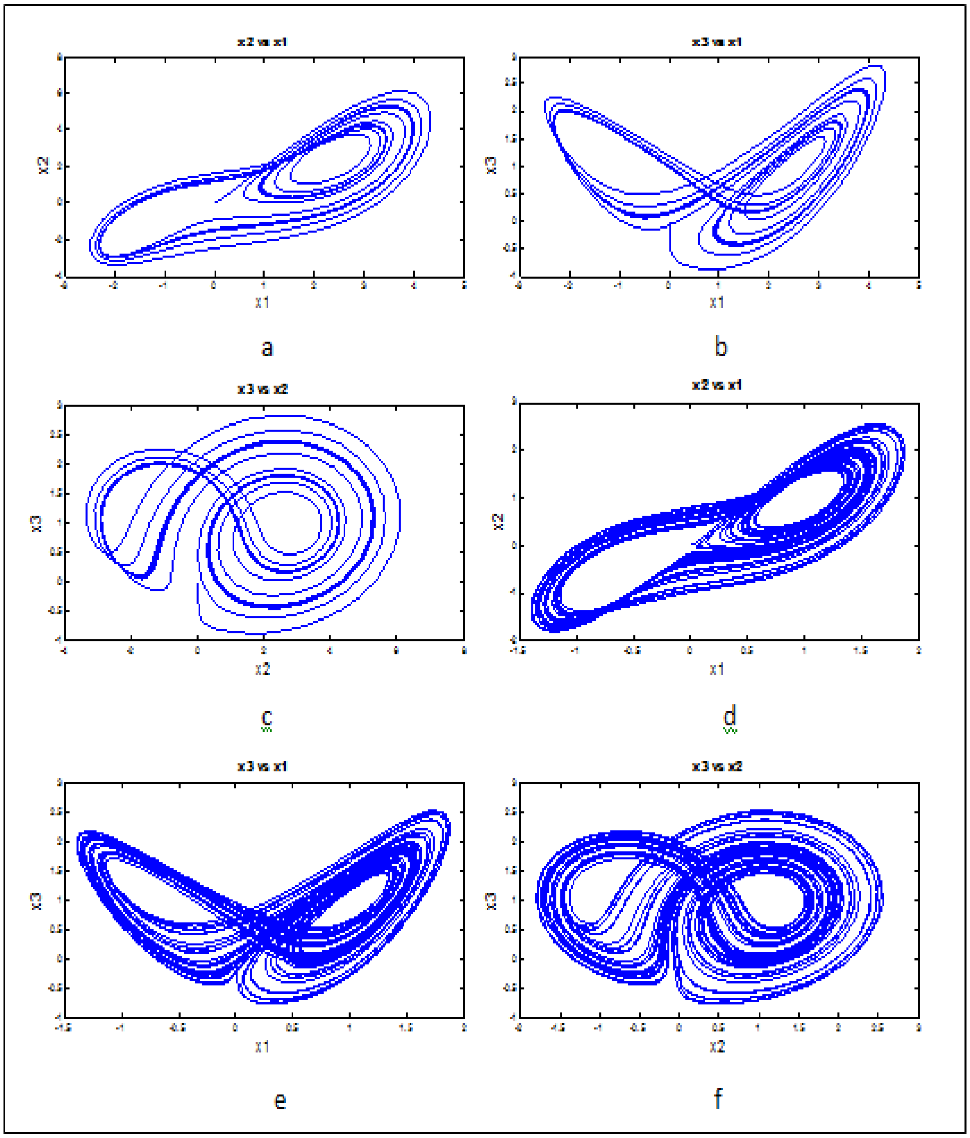

Figure 1. Evolution of strange attractors, (a) - (c), when the cross-product constant is $5x_1x_2$ and (d)-(f), when the constant is changed to $20x_1x_2$

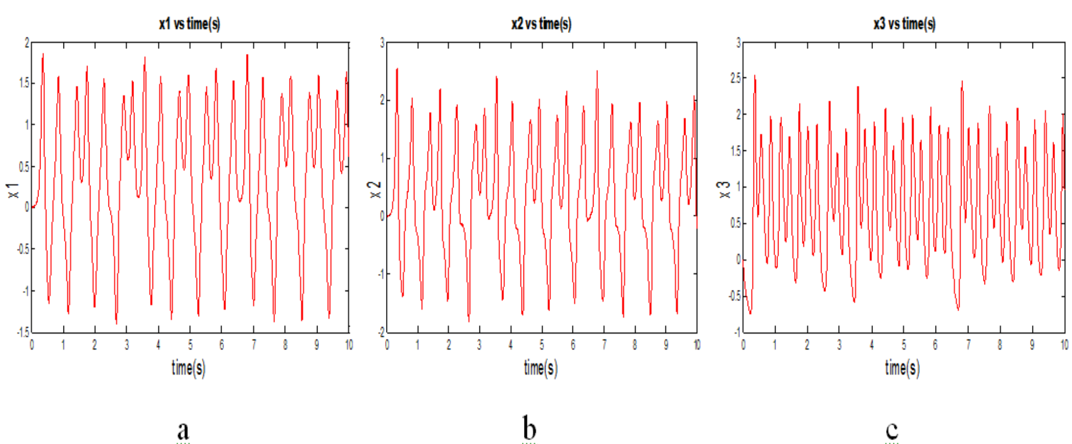

Figure 2. Time series evolution of the system trajectories

FUZZY MODELLING AND CONTROL

Fuzzy logic, on which fuzzy control evolved, is a computational paradigm on fuzzy set theory that allows for degrees of truth and falsehood. It provides a formal framework for constructing systems exhibiting both good numeric behavior and linguistic representation and can handle imprecise and incomplete information, thus making it possible to construct a model that can control a dynamic system without necessarily having a rigorous mathematical framework.. Mamdani (Mamdani, 1974) and Takagi Sugeno (TS) fuzzy models (Takagi and Sugeno, 1985), are widely used in modeling nonlinear systems. In the Mamdani fuzzy models, the antecedent and consequent parts of the fuzzy rules are fuzzy sets represented by membership functions, while the Takagi Sugeno model, the antecedent is a fuzzy set while the consequent part is a linear function of the input or a singleton. The TS fuzzy model is computational more efficient and has been utilized extensively in modelling nonlinear systems whose mathematical representations are rather intractable. More specifically, a generic and simplified form of the TS fuzzy model for an i-th fuzzy rule can be written in the following form given by:

$$\text{If } x \; is \; M_1 \; THEN \quad y = Ax + B \tag{6}$$

Where x is the input, M_1 is a linguistic variable, y is the output, A and B are constants.

The fuzzy controller synthesis was performed using Lyapunov stability theorem. A typical fuzzy logic controller architecture is given in Figure 3.

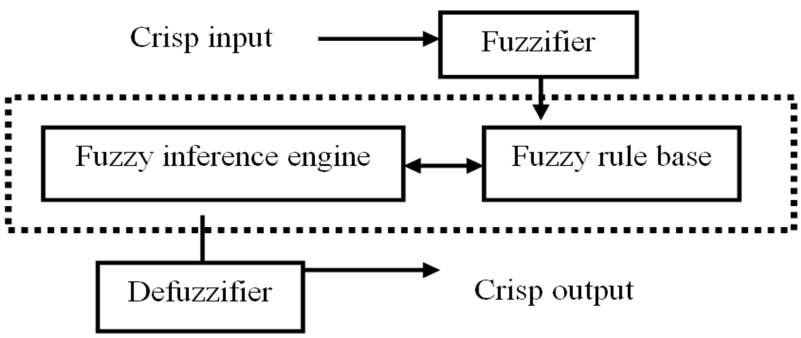

Figure 3. Fuzzy Logic Controller architecture

STABILITY AND STABLIZATION

Many studies on the stabilization of chaotic systems using fuzzy controllers or a hybrid of fuzzy and neural controllers have appeared in the literature (Precup et al., 2007; Wang and Ge, 2001; Vaidyanathan, 2012). In the work of Rezaie et al., (2006), Mamdani fuzzy controllers were used to stabilize the unstable periodic orbits of chaotic systems with satisfactory performances, although the number of rules was comparably large. In the framework of the Takagi-Sugeno fuzzy model (Takagi and Sugeno, 1985), an approach utilizing linear matrix inequality and so-called parallel distributed compensation was proposed by Tanaka and Wang (2001). This approach appeared appealing, however, the design approach is mathematically rigorous and the designed controllers required a large effort to stabilize the chaotic dynamics. In recent times, the Lyapunov stability theory has been studied in detailed and applied to the fuzzy stabilization problem (Wong et al, 2000). In the Lyapunov approach, a common Lyapunov function which guarantees asymptotic stability based on the dynamics of a given system is usually formulated. If the derivative of the Lyapunov function as a function of time is negative definite or negative semi-definite, then a sufficient condition for stability has been provided. An improved stability method proposed

ISSN-L: 2223-9553, ISSN: 2223-9944
Vol. 4 No. 4 July 2013

by Wong et al., 2000 has successfully applied to stabilize chaotic systems (Precup et al., 2007). This method which is based on the Takagi-Sugeno fuzzy modelling is used in this study.

Lyapunov Stability Criteria

The Lyapunov stability criteria provide a flexible and superb approach to analyzing the stability scenarios in a nonlinear system. These criteria have been outlined lucidly in various texts (Slotine and Li, 1991; Khalil, 2002). For the sake of clarity, we will repeat the universal definitions and theorems here. Consider a dynamic system which satisfies

$$\dot{x} = f(x,t), \ \ x(t_0) = x_0, \ \ x \in \Re^n \qquad (7)$$

Where $f: D \rightarrow \Re^n$ is a locally Lipschitz map from a domain $D \subset \Re^n$ into $\Re^n$. The following definitions and theorems therefore hold:

Definition (Khalil, 2002): the equilibrium point $x = 0$ of (7) is:

1. Stable if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such

$$\|\boldsymbol{x}(\mathbf{0})\| < \boldsymbol{\delta} \Rightarrow \|\boldsymbol{x}(\boldsymbol{t})\| < \boldsymbol{\varepsilon} \ , \forall \boldsymbol{t} \geq \mathbf{0} \qquad (8)$$

2. Unstable if it is not stable
3. Asymptotically stable if it is stable and δ can be chosen such that

$$\|\boldsymbol{x}(\mathbf{0})\| < \delta \Rightarrow \mathbf{lim}_{t\rightarrow\infty} \boldsymbol{x}(\boldsymbol{t}) = \mathbf{0} \qquad (9)$$

Selecting a Lyapunov function $V: D \rightarrow R$ such that

$$V(0) = 0 \text{ and } V(x) > 0 \text{ in } D - \{0\} \qquad (10)$$

$$\dot{V}(x) \leq 0 \quad \text{in } D - \{0\} \qquad (11)$$

Implying that $x = 0$ is stable in the sense of Lyapunov. Moreover, if

$$\dot{V}(x) < 0 \quad \text{in } D - \{0\} \qquad (12)$$

Then $x = 0$ is asymptotically stable.

To synthesize the Fuzzy Controller based on the method given by Wong et al., (1998), equation (12) must be negative definite in every fuzzy subsystem's active region and the defuzzification method given in equation (16) must be applied.

DESIGN OF THE FUZZY STABILIZATION CONTROLLERS

Given an autonomous nonlinear dynamic system comprising a plant and a Fuzzy Controller described by the equation

$$\dot{x} = f(x) + b(x)u, \ \ x(t_0) = x_0 \qquad (13)$$

Where $x = [x_1, x_2, \ldots \ldots x_n]\text{T}$ is a state vector, $f(x) = [f_1(x), f_2(x) \ldots \ldots f_n(x)]\text{T}$ and $b(x) = [b_1(x), b_2(x) \ldots \ldots b_n(x)]\text{T}$ are function vectors describing the dynamics of the plant, u is a control signal generated by the FLC. The FLC consists of p rules. The overall control signal applying to the plant is a function of u_i and Ξ_i (Wong, 2000) where u_i is the control signal generated by each fuzzy subsystem formed by the fuzzy rules. The i-th fuzzy rule of the Fuzzy Controller is of the following form:

Rule $\boldsymbol{i}$: IF $\boldsymbol{x_1}$ is Γ_{i1} AND $\boldsymbol{x_2}$ is Γ_{i2} $AND \ldots\ldots AND$ x_n is $\Gamma_{in,}$

THEN $u = u_i(x)$ (14)

Where $\Gamma_{i1}, \Gamma_{i2} \dots \Gamma_{in,}$ are inputs fuzzy labels, and $u = u_i(x)$ is the control output. Moreover, each fuzzy rule therefore generates a degree of fulfillment $\Xi_i(x)$ given by:

$$\Xi_i = \min(\Xi_{i1}, \Xi_{i2} \dots \Xi_{in}); \quad \Xi_i \in [0,1], \ i = 1,2 \dots n \qquad (15)$$

Definition (Wong et al., 2000): A fuzzy subsystem associated with fuzzy rule i is a system with a plant of (13) controlled by only u_i, which is the output of fuzzy rule i in the form of (14).

By using the singleton fuzzifier in conjunction with min-max inference and the weighted sum defuzzification method, the overall Fuzzy Controller output control signal is given by

$$U = \frac{\sum_{i=1}^{p} \Xi_i(x) u_i(x)}{\sum_{i=1}^{p} \Xi_i(x)} \qquad (16)$$

The fuzzy control scheme, triangular membership functions with linguistic terms Negative (N), Zero (Z) and Positive (P) chosen heuristically for state variables, x and y, with universe of discourse $\cup_{x_1} \in [-100,100]$ *and* $\cup_{x_2} \in [-80, 80]$; $\cup_{x_3} \in \emptyset$, and the fuzzy rule base are given in Figures 4 and 5 and Table 1 respectively.

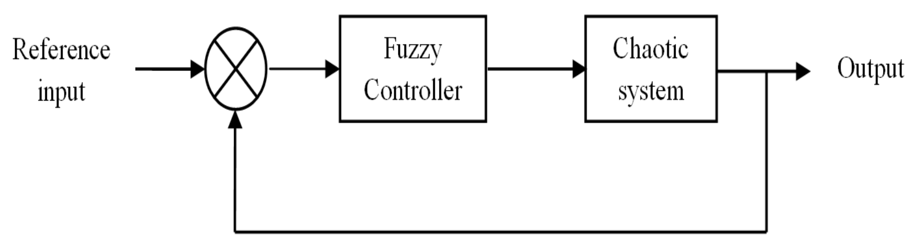

Figure 4. Fuzzy logic control scheme

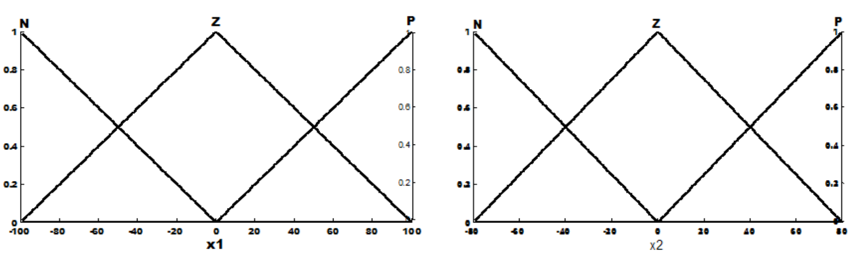

Figure 5. Triangular memberships function for x_1, x_2 ; $x_1 \in [-100\ 100]$; $x_2 \in [-80\ 80]$; $x_3 \in \emptyset$

Table 1. Fuzzy Controller rule base

Rule	*Antecedent*		*Consequent*
	x_1	x_2	u_i
1	P	P	u_1
2	N	N	u_2
3	P	N	u_3
4	N	P	u_4
5	P	Z	u_5
6	N	Z	u_6
7	Z	P	u_7
8	Z	N	u_8
9	Z	Z	u_9

FUZZY CONTROLLER SYNTHESIS

Theoretical results

Application of the Lyapunov stability criteria to synthesize the fuzzy controller requires the addition of a control input u to the original system equation and subsequent transformation into a state-space form (Precup et al., 2007). A detailed proof of the stability approach can be found in Wong et al., 1998, 2000). In this work, the control input was added to the first equation in (3) as given in (17)

$$\dot{x_1} = \sigma(x_2 - x_1) - x_2 - x_1 + u$$
$$\dot{x_2} = rx_1 - x_2 - 20x_1x_3 + x_1 + ax_2$$
$$\dot{x_3} = 5x_1x_2 - \beta x_3 + b + x_1(x_3 - c) \qquad (17)$$

The standard Lyapunov function was chosen

$$V^j(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \qquad (18)$$

$$j = 1,2..p, p = 9$$

(p is number of fuzzy rules).

The partial derivative of (18) yields

$$\dot{V}^j(x_1, x_2, x_3) = x_1\dot{x_1} + x_2\dot{x_2} + x_3\dot{x_3} \qquad (19)$$

Inserting (17) in (19) yields

$$\dot{V}^j(x_1, x_2, x_3) = x_1(\sigma(x_2 - x_1) - x_2 - x_1 + u) + x_2(rx_1 - x_2 - 20x_1x_3 + x_1 + ax_2) + 5x_1x_2 - \beta x_3 + b + x_1(x_3 - c)) \qquad (20)$$

Collecting like terms and taking $x_3 = 0$ (x_3 is an empty set)

$$\dot{V}^j(x_1, x_2, x_3) = x_1x_2(\sigma + r) - x_1^2(\sigma + 1) - x_2^2(1 - a) + x_1u \qquad (21)$$

In order to transform (21) into the form of (13)

Let $F = x_1x_2(\sigma + r) - x_1^2(\sigma + 1) - x_2^2(1 - a)$ (22)

$$B = x_1 \qquad (23)$$

$$\dot{V}^j(x_1, x_2, x_3) = F + B(u) \leq 0, j = 1,2 \dots 9 \qquad (24)$$

(F and B are scalar). From (24), the control signal applied to each fuzzy subsystem is therefore

$$u_j \geq -\frac{F}{B}, \ j = 1.2 \dots 9 \qquad (25)$$

By utilizing the fuzzy rule base table and inserting the parameters in (22) and (23), equation (24) was satisfied for all fuzzy subsystems. As a result, the trajectories were locally and globally stabilized in the sense of Lyapunov.

Numerical Simulation Results

The closed loop system was simulated with MATLAB for two scenarios i.e. with constants of nonlinearity $5x_1x_2$ and $20x_1x_2$ respectively.

Case 1: Using (3) when the constant is $5x_1x_2$ with initial conditions $[x_1(0), x_2(0), x_3(0)] = [0.01, 0.01, 0.1]$

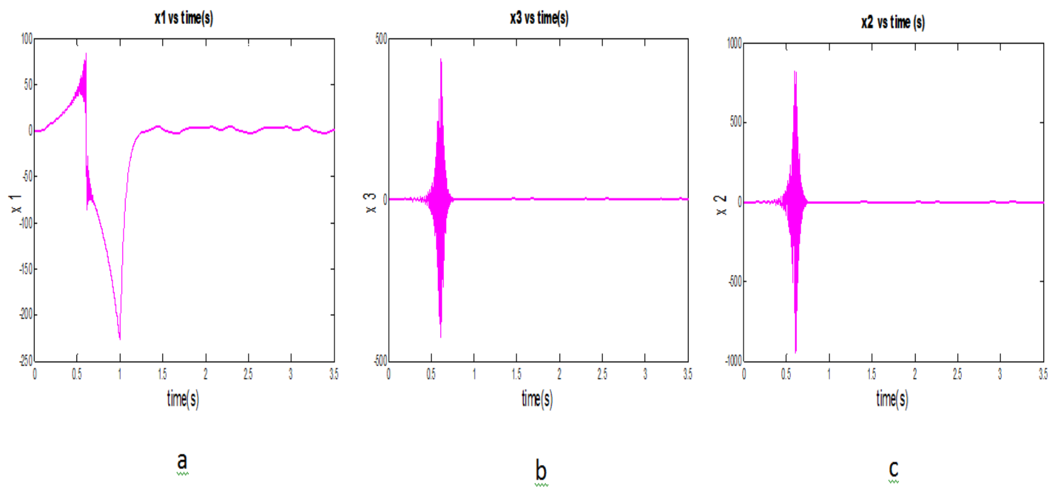

Figure 6. Controlled trajectories of the closed loop system for $5x_1x_2$

Case 2: When the constant associated with the nonlinearity x_1x_2 of the second equation of (3) is changed to $20x_1x_2$ for the same initial conditions as in case 1.

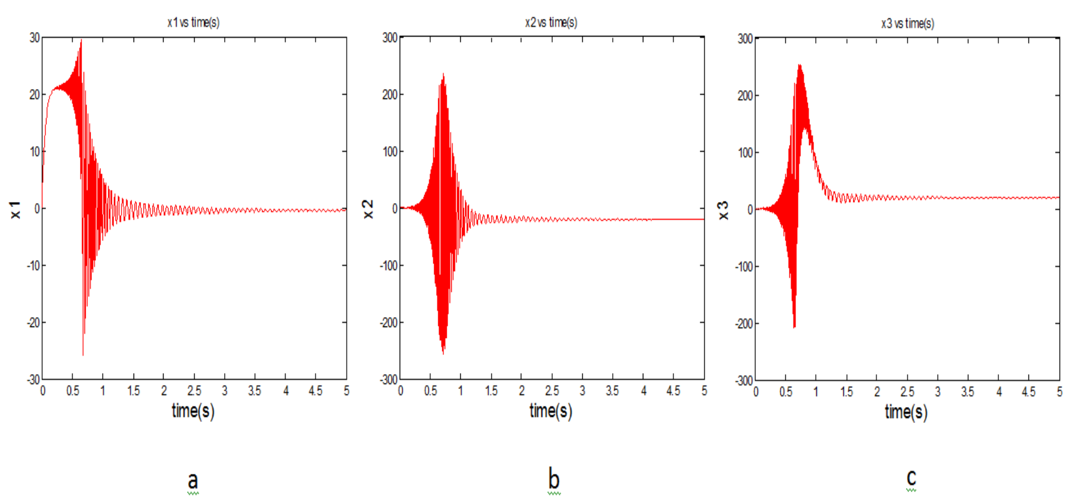

Figure 7. Controlled trajectories of the closed loop system for $20x_1x_2$

DISCUSSION

The Lorenz-Rossler system has rich dynamics which are topologically distinct from both Lorenz and Rossler systems. The results of simulations show that the system is stabilizable. However, the novel system presents more challenge to asymptotic stabilization than the same scenarios involving the individual Lorenz and Rossler systems. As shown in Figure 6 of Case 1, the system was asymptotically stabilized at some equilibrium points. However, in Case 2 where the system was stabilized in the neighborhoods of the origin.

CONCLUSION

This work shows that the trajectories of Lorenz-Rossler system are stabilizable. As the system is further studies, its rich dynamics can be further explored with the possibility of robust stabilizability. These possibilities assure the system's usefulness in synchronization of signals in secure communications applications.

ISSN-L: 2223-9553, ISSN: 2223-9944
Vol. 4 No. 4 July 2013

REFERENCES

Alsafasfeh, Q. H. & Al-Arni, M. S. (2011). A new chaotic behaviour from Lorenz and Rossler systems and its electronic circuit implementation. *Circuits and System*, 2, 101-105.

Cuomo, K. M., Oppenheim, A.V. & Strogatz, H. S. (1993). Synchronization of Lorenz-based chaotic circuits and application to secure communication, *IEEE Transaction on Circuits and Systems-II: Analogue and Digital Processing, 40*(10), 626-633.

Guan, X. P., Fan, Z. P. & Chen, C. L. (2002). *Chaos control and application in secure communication*. Beijing: National Defense Industry Press.

Guegan, D. (2009). Chaos in economics and finance. *Annual Reviews in Control, 33*(1), 89-93.

Khalil, H. K. (2002). *Nonlinear systems* (3rd edition). New Jersey: Prentice-Hall.

Kumar, A. & Hegde, B .M. (2012). Chaos theory: impact on and applications in medicine. *Nitte University Journal of Health Science, 2*(4), 93-99.

Lorenz, E.N. (1963). Deterministic nonperiodic flow. *Journal of Atmospheric Sciences, 20*, 130-141.

Lu, J., Chen, G., Cheng, D. & Celikovsky, S. (2002). Bridging the gap between the Lorenz system amd the Chen's system. *International Journal of Bifurcation and Chaos, 12*(12), 2917-2926.

Lu, J., Chen, G., Cheng, D (2004). A new chaotic system and beyond: the generalized Lorenz-like system." *International Journal of Bifurcation and Chaos, 14*(5), 1507-1537.

Mamdani, E. H. (1974). Application of fuzzy algorithm for control of a simple dynamic plant. *Proceedings of the IEEE*, 121, 1585-1588.

Pelivan, I. & Uyaroglu, Y. (2010). A new attractor from the general Lorenz system family and its electronic experimental implementation. *Turkish Journal of Electronic Engineering and Computer Science, 10*(2), 171-184.

Precup, R. E., Tomescu, M. L. & Preitl, S. (2007). Stabilization of Lorenz system using Fuzzy controllers. *International Journal of Computers, Communications and Control, 2*(3), 279-287, 2007

Rezaie, B., Cherati, Z. R., Motlagh, M. R. J. & Farrokhi, M. (2006). Stabilization of the Lorenz chaotic equations by fuzzy controllers. *Proceedings of the WASET, 15*, 246-249.

Robertson, R. & Combs, A. (1995). *Chaos theory in psychology and the life sciences*. New Jersey: Lawrence Erlbaum Associates, 1995

Slotine, J. J. E. & Li, W. (1991). *Applied nonlinear control*, New Jersey: Prentice-Hall.

Takagi, T. & Sugeno, S. (1985). Fuzzy identification of systems and its application to modeling and control. *IEEE Transactions on System, Man and Cybernetics, 15*, 116-132.

Tanaka, K. & Wang, H. O. (2001). *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: John Wiley.

Vaidyanathan, S. (2012). Output regulation of Sprott-F chaotic system by state feedback control. *International Journal of Control Theory and Computer Modelling, 2*(2), 53-64.

Wang, C. & Ge, S. S. (2001). Adaptive backstepping control of uncertain Lorenz system. *International Journal of Bifurcation and Chaos, 11*(4),1115-1119.

Wong, L. K., Leung, F. H. F. & Tam, P. K. S. (1998). Lyapunov function-based design of fuzzy controllers and its application on combining controllers." *IEEE Transaction on Industrial Electronics, 45*(3), 502-509.

Wong, L. K., Leung, F. H. F. & Tam, P. K. S. (2000). An improved Lyapunov function-based stability analysis method for fuzzy logic control systems. *Electronics Letters, 36*(12), 1085-1096.