FREE VIBRATION ANALYSIS OF THIN RECTANGULAR ISOTROPIC CCCC PLATE USING TAYLOR SERIES FORMULATED SHAPE FUNCTION IN GALERKIN'S METHOD

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ABSTRACT

In this study, Taylor series peculiar shape functions for CCCC isotropic thin rectangular plates were used on Galerkin's functional to determine the fundamental frequency of the plate under vibration. The study involved a theoretical derivation of peculiar shape function by applying the boundary conditions of the plate on Taylor series form of the plate equation. This initial result of the formulation was substituted on Galerkin's functional to obtain equation for the fundamental frequency of the free vibrating plate. Non – dimensional frequency parameter "k" was determined for different aspect ratios from 0.1 to 2.0 at an incremental rate of 0.1 respectively. The effectiveness of this method was demonstrated by comparing the solution obtained in the present study with existing approximate solution obtained from previous studies. The result as contained in table 2 show differences ranging from -0.0475% to 6.0557% which are acceptable in statistics. Thus, we can conclude that the present method is a good approximate method for analyzing plates in vibration.

Keywords: Boundary Conditions, Free Vibration, Fundamental Natural Frequency, Galerkin's Functional, Peculiar Shape Function, Taylor Series

INTRODUCTION

Vibrations are mechanical oscillations about an equilibrium position. Most of the time, the vibration of mechanical/structural systems are undesirable and eliminating or controlling vibrations may save human lives. Plates are structural elements that are frequently subjected to vibration and controlling the frequency at which plate vibrates is very important to structural designers. Free vibration analysis of thin rectangular plates that are clamped on all edges have been studied by many researchers in the past with the aim of calculating it's natural frequencies and this they have done using numerical approaches (Lee, (2004); Shi, (1990); Werfalli and Karoud, (2005) and Misra, (2012)) and energy variational methods (Lal et al, (2009); Shu et al, (2007); Sakata et al, (1996)). None of the existing solutions from past works used Taylor series formulated shape functions. It is worth mentioning that their approaches are very rigorous. The free vibration of thin plates is characterized by a fourth order partial differential equation and getting exact solutions for thin plates with all edges clamped might be difficult. This may be the reason why most of the reported solutions in literature were based on numerical and variational methods. The Galerkin's method is a very powerful, easy to comprehend and effectively relevant to the spectrum of engineering problems. The main purpose of this present study is to use the Taylor series formulated shape function in the Galerkin's functional to analyze free vibration of thin rectangular isotropic plate that is clamped on all its edges (CCCC).

GOVERNING DIFFERENTIAL EQUATION FOR THIN PLATE IN VIBRATION

The equation that delineates the flexural vibration of thin isotropic rectangular plates was derived by Njoku (2013) from the principles of the theory of elasticity and expressed as

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$$D(\frac{\partial^4 w_{(x,y)}}{\partial x^4} + \frac{2\partial^4 w_{(x,y)}}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{(x,y)}}{\partial y^4}) = \overline{m}\lambda^2 w_{(x,y)} \tag{1}$$

Where w is displacement in positive z – direction, $\overline{m}$ is the mass per unit area of the plate, λ is the fundamental natural frequency. The flexural rigidity is expressed as

$$D = \frac{Eh^3}{12(1-v)} \tag{2}$$

E = modulus of elasticity, h = plate thickness, v = Poisson's ratio.

Equation (1) can be represented in the form expressed as

$$\nabla^4 w_{(x,y)} - \overline{m}\lambda^2 w_{(x,y)} = 0 \tag{3}$$

$where$ ∇^4= Biharmonic differential operator and $\overline{m}\lambda^2 = inertia\ force$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{2\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{4}$$

GALERKIN'S FUNCTIONAL FORMULATION

Ventsel and Krauthmmer (2001), gave Galerkin's expression as

$$\iint [L(W_N) - F]\, f_i(x,y)\ dxdy = 0 \tag{5}$$

Expanding equation (5), we have,

$$\iint \left(L(W_N) f_i(x,y)\right) dxdy - \iint \left(F.f_i(x,y)\right) dxdy = 0 \tag{6}$$

Where L (W_N) is equal to $D\nabla^4$w, F is inertia force and f_i(x,y) is the trial function.

Substituting equation (3) into equation (6), we have,

$$D \iint (\nabla^4 w) f_i(x,y)\, dxdy - \overline{m}\lambda^2 \iint f_{i\,(x,y)}\, dxdy = 0 = G \tag{7}$$

In dimensionless co-ordinate system of R – Q axes, equation (7) can be expressed as

$$G = 0 = \frac{AD}{a^2} \iint \left(\frac{P\partial^4 f_{i\,(x,y)}}{\partial R^4} . f_{i\,(x,y)} + \frac{2\partial^4 f_{i\,(x,y)}}{P\partial R^2 \partial Q^2} . f_{i\,(x,y)} + \frac{\partial^4 f_{i\,(x,y)}}{P^3 \partial Q^4} . f_{i\,(x,y)}\right) \partial R \partial Q - APa^2\lambda^2 \overline{m} \iint f_{i(x,y)}^2\, \partial R \partial Q \tag{8}$$

Where P is aspect ratio, R and Q are non-dimensional axis parallel to x and y axis respectively.

$P = {}^{b}/_{a},\ R = {}^{x}/_{a}\ \ Q = {}^{y}/_{b}$ and W = A. f_i

Equation (8) is the Galarkin's functional in dimensionless parameters R & Q, and aspect ratio P = b/a.

CCCC PLATE BOUNDARY CONDITIONS

Defection and slope vanishes along the clamped edges. Figure 1 shows a rectangular plate that is clamped on all edges in dimensionless co-ordinate system.

R

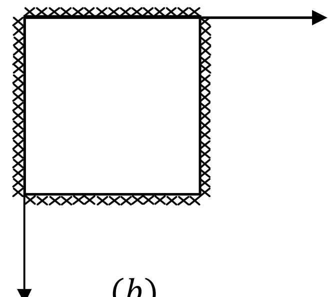

Figure 1: CCCC plate in dimensionless co-ordinate system of R – Q axes

CCCC plate's boundary conditions are:

$$w(R=0) = w'^{R}(R=0) = 0 \quad (9)$$
$$w(R=1) = w'^{R}(R=1) = 0 \quad (10)$$
$$w(Q=0) = w'^{Q}(Q=0) = 0 \quad (11)$$
$$w(Q=1) = w'^{Q}(Q=1) = 0 \quad (12)$$

Where w'^{Q} & w'^{R} are the first derivative of the displacement functions in Q and R axis respectively.

TAYLOR SERIES FORMULATED SHAPE FUNCTION

Ibearugbulem in 2012, using Taylor Mclaurin's series formulated the general shape function for thin rectangular plates as;

$$w = \sum_{m=0}^{4} \sum_{n=0}^{4} a_m b_n R^m Q^n \quad (13)$$

Substituting equations (9) and (11) into equation (13) we have:

$$a_0 = 0\,;\ a_1 = 0\,;\ b_0 = 0\,;\ b_1 = 0$$

Also, substituting equation (10) into equation (13) and solving the resulting two simultaneous equations we have:

$$a_2 = a_4 \; and \; a_3 = -2a_4$$

Similarly, substituting equation (12) into equation (13) and solving the resulting two simultaneous equations, we have:

$$b_2 = b_4 \; and \; b_3 = -2b_4$$

Substituting the values of $a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, and\ b_4$ into equation (13) gave a peculiar shape function for CCCC plate as

$$W = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)\ a_4 b_4 = A.f_1 \quad (14)$$

Where:

$$f_1 = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad and\ \ a_4 b_4 = A \quad (15)$$

FORMULATION OF THE FUNDAMENTAL NATURAL FREQUENCY EQUATION FOR CCCC PLATE

If $f_1 = f_i$ in equation (8), the partial derivatives of equation (15) with respect to R or Q or both gave the following equations,

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Vol. 4 No. 4 July 2013

$$\frac{\partial^4 f_1}{\partial Q^4} = 24(R^2 - 2R^3 + R^4) \tag{16}$$

$$\frac{\partial^4 f_1}{\partial R^4} = 24(Q^2 - 2Q^3 + Q^4) \tag{17}$$

$$\frac{\partial^4 f_1}{\partial R^2 \partial Q^2} = 4(1 - 6R + 6R^2)(1 - 6Q + 6Q^2) \tag{18}$$

$$\frac{\partial^4 f_1}{\partial R^4} . f_1 = 24(R^2 - 2R^3 + R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \tag{19}$$

$$\frac{\partial^4 f_1}{\partial Q^4} . f_1 = 24(Q^2 - 2Q^3 + Q^4)(R^4 - 4R^5 + 6R^6 - 4R^7 + R^8) \tag{20}$$

$$\frac{\partial^4 f_1}{\partial R^2 \partial Q^2} . f_1 = 4(R^2 - 8R^3 + 19R^4 - 18R^{15} + 6R^6)(Q^2 - 8Q^3 + 19Q^4 - 18Q^{15} + 6Q^6) \tag{21}$$

$$f_i^2 = (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \tag{22}$$

Integrating them partially with respect to R and Q gave the following:

$$\int_0^1 \int_0^1 \frac{\partial^4 f_1}{\partial R^4} . f_1 \partial R \partial Q = 0.00127 \tag{23}$$

$$\int_0^1 \int_0^1 \frac{\partial^4 f_1}{\partial Q^4} . f_1 \partial R \partial Q = 0.00127 \tag{24}$$

$$\int_0^1 \int_0^1 f_1^2 \, \partial R \partial Q = 0.0000025195 \tag{25}$$

$$\int_0^1 \int_0^1 \frac{\partial^4 f_1}{\partial R^2 \partial Q^2} . f_1 \partial R \partial Q = 0.00036 \tag{26}$$

Substituting equations (23), (24), (25) and (26) into equation (8), we have:

$$\frac{DA}{a^2}\left(0.00127P + \frac{0.00072}{P} + \frac{0.00127}{P^3}\right) = APa^2\lambda^2(0.0000025195\overline{m}) \tag{27}$$

Making λ^2 the subject of the gave

$$\lambda^2 = \frac{D}{Pa^4\overline{m}}\left(504.0683P + \frac{285.7143}{P} + \frac{504.0683}{P^3}\right) \tag{28}$$

$$\text{Let } \mathrm{K}^2 = \left(504.0683 + \frac{285.7143}{P^2} + \frac{504.0683}{P^4}\right) \tag{29}$$

K is the non-dimensional frequency parameter.

We have:

$$\lambda = \frac{\mathrm{K}}{a^2} * \sqrt{\frac{D}{\overline{m}}} \tag{30}$$

Equation (30) is the fundamental natural frequency equation for CCCC plate for aspect ratio P = b/a.

RESULTS AND DISCUSSION

The non-dimensional frequency parameter, 'K' was computed for aspect ratios ranging from 0.1to 0.2 at increment of 0.1 as presented in table 1. This was done by substituting these values respectively into equation (29). In order to validate the solutions of this present study, comparison was made with solutions from previous research works [(Liew et al (1995), Sakata et al (1996) and Chakraverty (2009)] as shown in table 2.

Table 1. Non-dimensional frequency parameter (K) for aspect ratios from present study (P = a/b)

P	*K*
0.1	2251.6139
0.2	568.059
0.3	256.7293
0.4	148.2576
0.5	98.5507
0.6	72.0229
0.7	56.4507
0.8	46.7035
0.9	40.3132
1.0	35.9709
1.1	32.9322
1.2	30.7508
1.3	29.1488
1.4	27.9479
1.5	27.0305
1.6	26.3175
1.7	25.7547
1.8	25.3039
1.9	24.9381
2.0	24.6377

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Table 2. Non-dimensional frequency parameter (K) of thin rectangular plates with all edges clamped (CCCC) for aspect ratio of b/a

P	Present study	Liew et al (1995)	Sakata et al (1996)	Charkraverty (2009)	% Difference b/w liew et al(1995) and present study	% Difference b/w sakata et al(1996) and present study	% Difference b/w charkraverty et al(1995) and present study
0.4	148.2576	147.7677	-	_	0.3315	-	-
0.5	98.5507	-	95.391	98.317	-	3.3124	0.2378
0.6	72.0229	71.9139	-	-	0.1516	-	-
1.0	35.9709	-	33.917	35.988	-	6.0557	-0.0475
2.0	24.6377	-	23.848	-	-	3.3114	-

Comparing the present solution with the solution given by Chakraverty (2009) for aspect ratios of 0.5 and 1, it was found that the present solutions are very close to Chakraverty (2009) solutions as the percentage differences between them are 0.2378% and –0.0475% respectively. Also the percentage difference between the solutions given by Liew et al (1995) and the present solutions for aspect ratios 0.4 and 0.6 are 0.3351% and 0.1516% which means that their solutions are close to one another. Hence, from these two comparisons, it can be seen that the assumed shaped function is close to the exact shape function for a plate that is clamped on all edges however, the solution given by Sakata et al (1996) for aspect ratio of 1 gave the highest percentage difference of 6.0557% when compared with that from present study, but this is also within the acceptable range in statistics. Hence, we can conclude that the present method is a good approximate method for analyzing plates in vibration.

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