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THE EQUATIONS FOR ESTIMATING OIL PRODUCTION BEFORE, AT AND AFTER BREAKTHROUGH FROM A RESERVOIR PRODUCED BY FLUID INJECTION

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ABSTRACT

Estimation of oil production from a reservoir being produced by fluid injection was presented. Equations were propounded for estimating oil production before and at breakthrough from a reservoir being produced by fluid injection. The propounded equations are based on the rules of expressing viscosities and energy in terms of pressure and volume. The applicability of the equations was based on certain assumptions which were listed in the Theory. One of the Oil Production Equations was actually applied to compute the volume of oil producible from Reservoir AGU-2, located South-East Nigeria, before breakthrough. A Microsoft visual basic program was also developed based on the Oil Production Equation that can estimate oil recovery before breakthrough during a fluid injection process.

Keywords: Oil, production, recovery, fluid, injection, equation, viscosity, volume, breakthrough, reservoir

INTRODUCTION

According to Tharek, (2001), six driving mechanisms basically provide the natural energy necessary for oil recovery:

1. Rock and liquid expansion
2. Solution gas drive
3. Gas cap drive
4. Water drive
5. Gravity drainage drive
6. Combination drive

The recovery of oil by any of the above driving mechanisms is called primary recovery. The term refers to the production of hydrocarbons from a reservoir without the use of any process (such as water injection) to supplement the natural energy of the reservoir. The primary drive mechanism and anticipated ultimate oil recovery should be considered when reviewing possible steam flooding prospects. The approximate oil recovery range is tabulated below for various driving mechanisms. Note that these calculations are approximate and, therefore, oil recovery may fall outside these ranges.

Generally, for oil to flow the primary drive mechanisms help to drive the oil to the surface. Soon the initial pressure of the reservoir drops below economic limits after the reservoir recovers at least 40% of the oil in place. Enhanced oil recovery helps to recover the remaining 60% locked in the subsurface (Muonagor and Nnakaihe, 2011). Figure 1 illustrates the different methods of enhanced oil recovery.

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Table 1. Approximate recovery range in reservoir with various drive mechanisms

Driving Mechanism	*Oil Recovery Range %*
Rock and liquid expansion	3–7
Solution gas	5–30
Gas cap	20–40
Water drive	35–75
Gravity drainage	<80
Combination drive	30–60

Source: Tharek, (2001)

Solution gas drive reservoirs and volumetric under saturated oil reservoirs especially are very good candidates for fluid injection as their natural oil recovery percentage is very low. Before any enhanced oil recovery project or precisely fluid injection is embarked on, the recovery calculations and estimations have to be performed.

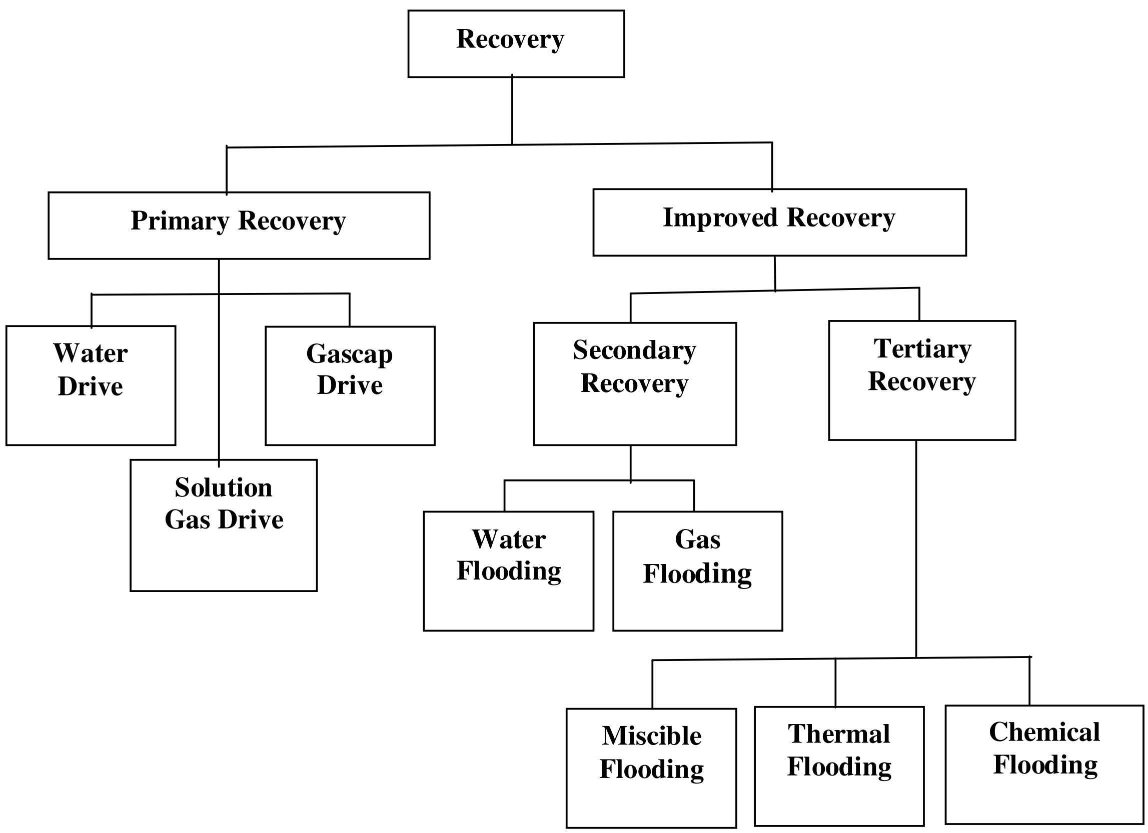

Figure 1. Different Methods of Enhanced Oil Recovery

Source: Muonagor and Nnakaihe, (2011)

There had not been any specific model or equation for computing oil recovery during all kinds of fluid injection, estimation of oil recovery was simply based on taking certain mathematical steps, hypothetical assumptions and speculations. This work introduces simple and direct equations for the computation of volume of oil recoverable from a reservoir before

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and at breakthrough during production by any kind of fluid injection. The equation is based on the rules of expressing viscosities and energy in terms of pressure and volume.

THEORY

In developing the equation for estimating oil production from a reservoir before, at and after breakthrough during Enhanced Oil Recovery by fluid injection, certain parameters are taken into consideration. The parameters are given below:

1. Volume of the injected fluid
2. Time for Injection and Production
3. Viscosity of the injection fluid
4. Injection rate
5. Produced oil formation volume factor
6. Flood area
7. Reservoir thickness
8. Reservoir Porosity
9. Viscosity of oil
10. Connate water saturation

Estimation of Oil Production before Breakthrough

From fluid mechanics, Energy = Pressure * Volume 1

Therefore Energy of Injection fluid, $E_i = P_i * V_i$ 2

And Energy of Produced fluid, $E_p = P_p * V_p$ 3

Where P_i, V_i, P_p and V_p are injection fluid pressure, injection fluid volume, produced fluid pressure and produced fluid volume respectively.

E_i must be greater than or equal to E_p for flow to occur ie condition for flow is given as:

$P_iV_i \geq P_pV_p$

When $P_iV_i = P_pV_p$, then V_i is the minimum volume of fluid (bbl) injected at pressure, P_i (psi) to cause a volume of V_p of oil (bbl) to be produced at pressure, P_p (psi).

According to Muonagor and Nnakaihe, (2011), volume of oil (bbl) is expressed as:

$V_{oil} = 7758Ah\Phi(1 - S_{wc})/B_o$ 4

Where B_o = Oil Formation Volume Factor, rb/stb

A = Flood Area, acre

h = Reservoir Thickness, ft

Φ = Reservoir Porosity

S_{wc} = Connate Water Saturation

Then, the minimum volume of injection fluid required to displace V_{oil} is given as:

$V_i = P_p * V_{oil} / P_i$ 5

Which is the same as:

$V_i = P_p * 7758Ah\Phi(1 - S_{wc}) / (P_i * B_o)$ 6

From dimensional analysis, viscosity could be expressed as:

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μ = (Force/Area)/((Length/Time) * (1/Time)) 7

Viscosity of the injection fluid (cp) could as well be expressed as:

$\mu_i = P_i/(\bar{U}_i/d_i)$ 8

where $\bar{U}_i$ = injection fluid velocity

d_i = injection well depth

The injection fluid viscosity can also be expressed as:

$\mu_i = A_iP_i/(A_i\bar{U}_i/d_i)$ 9

where A_i = cross-sectional area of the injection path

$\mu_i = A_iP_i/(I_{rate}/d_i)$ 10

where i_{rate} = flow rate of injection fluid ie injection rate, bbl/day

If P_i is made the subject,

$P_i = (\mu_iI_{rate})/(A_id_i)$ 11

$P_i = (\mu_iV_i/t_i)/(A_id_i)$ 12

Where t_i = time taken in days for V_i volume of fluid to be injected

Then, P_p can also be expressed as:

$P_p = (\mu_oV_o/t_o)/(A_pd_p)$ 13

Where μ_o = produced oil viscosity

V_o = volume of oil produced

t_o = time taken for V_o volume of oil to be produced

A_p = production path cross-sectional area

d_p = depth of production well

Substitute eqs 11 and 13 into eq 6, volume of injection fluid becomes

$V_i = [(\mu_oV_o/t_o)/(A_pd_p) * 7758Ah\Phi(1 - S_{wc})] / [(\mu_iI_{rate})/(A_id_i) * B_o]$ 14

If injection and production wells have the same cross-sectional area and depth, $A_p = A_i$ and $d_p = d_i$

Then, $V_i = [(\mu_oV_o/t_o) * 7758Ah\Phi(1 - S_{wc})] / [(\mu_iI_{rate}) * B_o]$ 15

Making V_o the subject of the equation,

$V_o = [V_It_o\mu_II_{rate}B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]$ 16

At any time before breakthrough, the equation for estimating oil production from a reservoir during fluid injection is given below:

$V_o = [t_it_o\mu_I(I_{rate})^2B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]$ 17

Where V_o is in bbl; t_i and t_o in days; μ_I and μ_o in cp; I_{rate} in bbl/day; B_o in rb/stb; A in acre; h in ft;

Estimation of Oil Production at Breakthrough

According to Muonagor and Nnakaihe, (2011), fluid injected at breakthrough is expressed as:

$V_{iBT} = 7758Ah\Phi * E_{ABT} * Q_{IBT}$ 18

$V_{iBT} = I_{rate} * t_{iBT}$ 19

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Where E_{ABT} = areal sweep efficiency at breakthrough expressed as a function of mobility ratio, M, t_{iBT} = injection time to breakthrough and Q_{IBT} = pore volumes of fluid injected at breakthrough which is given below as:

$$Q_{IBT} = S_{fBT} - S_{fi} \quad (20)$$

Putting eqs 18 and 19 into eq 17, the propounded equation for estimating oil production;

$$V_{oBT} = [7758Ah\Phi(E_{ABT}Q_{IBT})^2 t_{oBT}\mu_I B_o] / [t_{iBT}\mu_o(1 - S_{wc})] \quad (21)$$

Where t_{oBT} = oil production time to breakthrough, day

V_{oBT} = volume of oil produced at breakthrough, bbl

S_{fBT} = average injection fluid saturation at breakthrough

S_{fi} = initial injection fluid saturation

Estimation of Oil Production after Breakthrough

The cumulative oil production after breakthrough is given below as:

$$V_{oABT} = V_{oBT} + V_{of} - F_p \quad (22)$$

Where V_{of} = total subsequent fluid production after breakthrough, bbl

F_p = injection fluid production, bbl

Note that after breakthrough, injection fluid is produced along with the oil from the reservoir.

The total subsequent fluid production after breakthrough is expressed as:

$$V_{of} = [((t_{ABT} - t_{iBT}) * I_{rate})^2 \mu_I B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})] \quad (23)$$

Where t_{ABT} = the reference time after breakthrough, day

The injection fluid production is expressed as:

$$F_p = [V_{iBT} + V_{ABT} - F_t]/B_f \quad (24)$$

Where B_f = formation volume factor of injection fluid

V_{ABT} = volume of fluid injected from breakthrough time to t_{ABT}, bbl, also expressed as:

$$V_{ABT} = (t_{ABT} - t_{iBT}) * I_{rate} \quad (25)$$

F_t = volume of injected fluid trapped in the reservoir, bbl, also expressed mathematically as:

$$F_t = 7758Ah\Phi E_{AABT}(S_{ff} - S_{fi}) \quad (26)$$

Where S_{ff} = injection fluid saturation in the reservoir at time, t_{ABT}

E_{AABT} = areal sweep efficiency after breakthrough and it is given as:

$$E_{AABT} = E_{ABT} + 0.633Log[(V_{IBT} + V_{ABT})/V_{IBT}] \quad (27)$$

Putting eqs 21, 23, 24, 25 and 26 into eq 22:

$$V_{oABT} = [[7758Ah\Phi(E_{ABT}Q_{IBT})^2 t_{oBT}\mu_I B_o] / [t_{iBT}\mu_o(1 - S_{wc})]] + [[((t_{ABT} - t_{iBT}) * I_{rate})^2 \mu_I B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]] - [(7758Ah\Phi * E_{ABT} * Q_{IBT})/B_f] - [((t_{ABT} - t_{iBT}) * I_{rate})/B_f] + [(7758Ah\Phi E_{AABT}(S_{ff} - S_{fi}))/B_f] \quad (28)$$

Assumptions of the Oil Recovery Equations

I. The fluid is injected continuously
II. Injection fluid by-passing of oil is highly negligible
III. Injection and production wells have the same cross-sectional area and depth

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The summary of the new equations propounded in this work is shown in Table 2 below.

Table 2. Summary of the New Equations Propounded in this work.

Parameter	*Propounded Equation*
Minimum Volume of Injection Fluid Required to Displace Oil, bbl	$V_i = P_p * 7758Ah\Phi(1 - S_{wc}) / (P_i * B_o)$
Volume of Oil Production Before Breakthrough, bbl	$V_o = [t_i t_o \mu_I (I_{rate})^2 B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]$
Volume of Oil Production At Breakthrough, bbl	$V_{oBT} = [7758Ah\Phi(E_{ABT}Q_{IBT})^2 t_{oBT}\mu_I B_o] / [t_{iBT}\mu_o(1 - S_{wc})]$
Total Subsequent Fluid (Injected and Produced Fluids) Production After Breakthrough, bbl	$V_{of} = [((t_{ABT} - t_{iBT}) * I_{rate})^2 \mu_I B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]$
Volume of Injected Fluid Trapped in the Reservoir, bbl	$F_t = 7758Ah\Phi E_{AABT}(S_{ff} - S_{fi})$
Volume of Fluid Injected from Breakthrough Time to t_{ABT}, bbl	$V_{ABT} = (t_{ABT} - t_{iBT}) * I_{rate}$
Volume of Injected Fluid Produced After Breakthrough, bbl	$F_p = [V_{iBT} + V_{ABT} - F_t]/B_f$
Volume of Oil Production After Breakthrough, bbl	$V_{oABT} = V_{oBT} + V_{of} - F_p$
Volume of Oil Production After Breakthrough, bbl	$V_{oABT} = [[7758Ah\Phi(E_{ABT}Q_{IBT})^2 t_{oBT}\mu_I B_o] / [t_{iBT}\mu_o(1 - S_{wc})]] + [[((t_{ABT} - t_{iBT}) * I_{rate})^2 \mu_I B_o] / [7758Ah\Phi\mu_o(1 - S_{wc})]] - [(7758Ah\Phi * E_{ABT} * Q_{IBT})/B_f] - [((t_{ABT} - t_{iBT}) * I_{rate})/B_f] + [(7758Ah\Phi E_{AABT}(S_{ff} - S_{fi}))/B_f]$

A Microsoft visual basic program was developed using the equation for estimating oil production from a reservoir before breakthrough during fluid injection. The sample of the Microsoft visual basic program is shown in Figure 2 below.

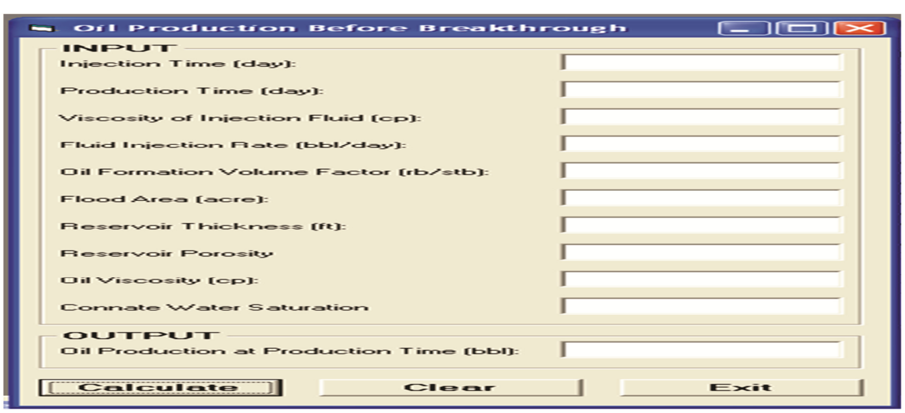

Figure 2. Microsoft Visual Basic Program for Oil Production Estimation before Breakthrough

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RESULT

Case: Reservoir AGU-2

Reservoir AGU-2 located in South-East Nigeria is a reservoir whose primary energy became depleted with consequent pressure reduction and has approached a limit where further production by the primary recovery methods is uneconomical and insufficient. The reservoir is to be produced with steam injection. The reservoir and fluid data are given in Table 3 below.

Table 3. Reservoir and Fluid Data for Reservoir AGU-2

Reservoir Pressure	*2300 psia*
Oil Viscosity	1 cp
Oil FVF	1.401 rb/stb
Reservoir Thickness	80 ft
Reservoir Porosity	0.2
Connate Water Saturation	0.2
Flood Area	40 acres
Oil Production Time	480 days

The steam data is presented in Table 4 below:

Table 4. Steam Data for the Injection of Steam into Reservoir AGU-2

Injection Time	*500 days*
Steam Viscosity	0.35
Steam Injection Rate	4600 bbl/day

The Microsoft visual basic program for estimating oil production using the Oil Production Equation is as shown in Figure 3 below.

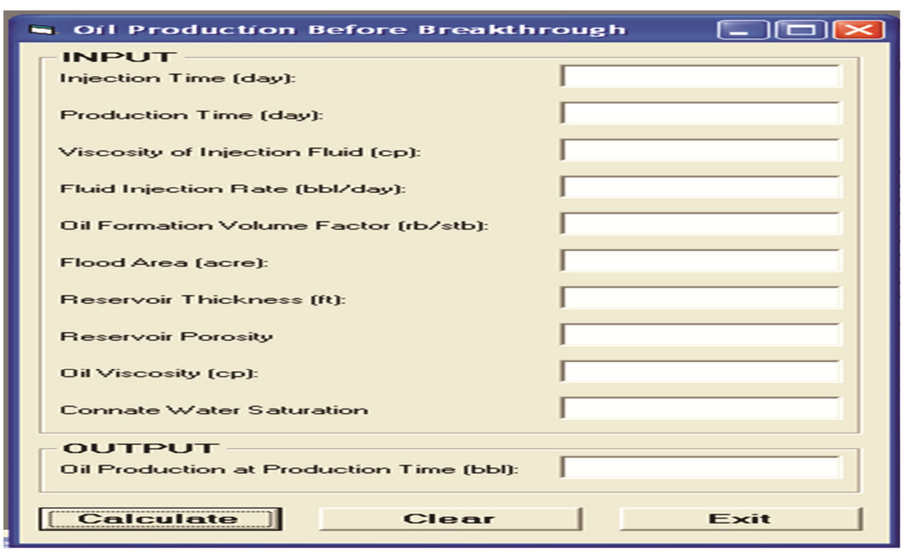

Figure 3: Microsoft Visual Basic Program for Reservoir AGU-2 Oil Production Estimation

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CONCLUSION

At the end of this work, it is shown that the volume of oil producible before, at and after breakthrough from a reservoir produced by fluid injection can be computed. These Oil Production Equations would serve great purpose as there had not been any previous equations for the computation of oil recovery from the reservoir by all fluids injection. The Oil Production Equations shown in equations 17, 21 and 28 are comprehensive and simple to apply once the necessary parameters are given. They give straight forward approaches to oil production calculations during production by fluid injection.

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NOMENCLATURE

A = Flood Area, acre

A_i = Cross-sectional Area of the Injection Path

A_p = Production Path Cross-sectional Area

bbl = Barrel of Fluid

B_o = Oil Formation Volume Factor, rb/stb

cp = Centipoise

d_i = Injection Well Depth

d_p = depth of production well

E_{ABT} = Areal Sweep Efficiency at Breakthrough

E_i = Energy of the Injection Fluid

E_p = Energy of Produced fluid

F_p = Injection Fluid Production, bbl

F_t = Volume of Injected Fluid Trapped in the Reservoir, bbl

ft = Foot

h = Reservoir Thickness, ft

I_{rate} = Fluid Injection Rate, bbl/day

M = Mobility Ratio

P_i = Injection Pressure

P_p = Produced Fluid Pressure

Q_{IBT} = Pore Volume of Fluid Injected at Breakthrough

rb = Reservoir Barrel of Fluid

stb = Stock Tank Barrel of Fluid

S_{wc} = Connate Water Saturation

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S_{fBT} = Average Injection Fluid Saturation at Breakthrough

S_{fi} = Initial Fluid Saturation

S_{ff} = Injection Fluid Saturation in the Reservoir at Time, t_{ABT}

t_{ABT} = The Reference Time after Breakthrough, day

t_{iBT} = Injection Time to Breakthrough

t_o = Time Taken for Vo Volume of Oil to be Produced

V_{oBT} = Oil Production at Breakthrough, bbl

V_{ABT} = Volume of Fluid Injected from Breakthrough Time to t_{ABT}, bbl

V_{oABT} = Cumulative Oil Production after Breakthrough, bbl

V_{of} = Total Subsequent Fluid Production after Breakthrough, bbl

V_i = Injection Fluid Volume

V_o = volume of oil produced

V_p = Produced Fluid Volume

V_{oil} = Oil Volume

V_{iBT} = Volume of Injected Fluid at Breakthrough

Φ = Reservoir Porosity

μ = Viscosity

$\bar{U}_i$ = Injection Fluid Velocity

μ_I = Viscosity of the Injected Fluid, cp

μ_o = Oil Viscosity, cp

psia = Pounds per Square Inch (atmosphere)

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