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STRESS ANALYSIS OF CONICAL SHELL

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ABSTRACT

This paper studies the dynamic stresses and deformation of stiffened conical shell and compared between the steady state and transient analysis. The same conical shells used in static analysis of conical shell were taken with suddenly applied pressure. The element used is a modified eight-node superparametric shell element. The effects of the number and cross-section area of stiffened shell are analyzed. The results are compared with available research and MSC\NASTRAN.

Keywords: Conical Shells, Dynamic Analysis, Finite Element

INTRODUCTION

The dynamic response of cylindrical and conical panels subjected to arbitrary time-varying load distributions was studied and appropriate equations were presented by Ross and Praise (Rossettos, 1969). Convenient trigonometric series were used, in conjunction with finite-difference methods, to reduce the governing equation to sets of matrix equations. The numerical solution procedure involves time integration using a Gaussian elimination technique particularly suited to the banded matrices involved. Calculated results treat the effects of conicity and various support conditions on the structural response. Individual cases show quantitatively how cylindrical panel response is more sensitive than that of conical panels to changes in edge restraint (Mannan *et al.,* 2009).

Dynamic analysis of stiffened conical shell was presented (Srinivasan, 1989). By using an integral equation method in the space domain. The smearing technique was used for closely spaced stiffeners. The time domain analysis had been done using the mode super position method. The effect of eccentricity of stiffeners, varying the cross-sectional dimensions and spacing of the stiffeners had been studied. The application of integral equation technique to dynamic response problem has been illustrated by considering the layered conical shell panel, which was introduced (Srinivasan, 1989). The method consists of using the integral equation technique in the space domain and direct integration using the Wilson-Theta method in the time domain since results were not available for layered conical shell panels.

This element consists of four corner and four mid side nodes. The nodal degrees of freedom considered are the three translations u, v, w of the mid surface and two rotations α and β of the normal to the mid surface. The Cartesian coordinates of any point of the shell and the curvilinear coordinates can be written in the form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum N_i \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}_{middle} + \sum N_i \frac{\zeta}{2} \begin{bmatrix} l_{3i} \\ m_{3i} \\ n_{3i} \end{bmatrix} h \qquad (1)$$

Where:

l_{3i}, m_{3i} and n_{3i} are the direction cosines of the normal to the middle surface. Here N_i is a function taking a value of unity at the node i and zero at all other nodes, it is called as "shape function ".

In the kinematics formulation two assumptions are imposed:

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1. Nodal fiber is inextensible.

2. Only small rotations are considered.

The displacements at any point (ξ, η, ζ) can be expressed in terms of the nodal displacements as

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^{8} N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} + \sum_{i=1}^{8} N_i \zeta \frac{h_i}{2} \mu_i \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (2)$$

In this formula the symbol μi denote the following matrix:

$$\mu_i = \begin{bmatrix} -l_{2i} & l_{1i} \\ -m_{2i} & m_{1i} \\ -n_{2i} & n_{1i} \end{bmatrix}$$

Column 1 in this matrix contains negative values of the direction cosines of the second tangential vector. The displacement shape functions may be cast into the matrix form:

$$[N_i]=[N_{Ai}]+\zeta\,[N_{Bi}] \quad (i=1,2\ldots8) \quad (3)$$

$$[N_{Ai}]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} N_i \quad \& \quad [N_{Bi}]=\begin{bmatrix} 0 & 0 & 0 & -l_{2i} & l_{1i} \\ 0 & 0 & 0 & -m_{2i} & m_{1i} \\ 0 & 0 & 0 & -n_{2i} & n_{1i} \end{bmatrix} \frac{h_i}{2} N_i \quad (4)$$

The 3 X 3 Jacobian matrix required in this formulation is:

$$[J]=\begin{bmatrix} x,\xi & y,\xi & z,\xi \\ x,\eta & y,\eta & z,\eta \\ x,\zeta & y,\zeta & z,\zeta \end{bmatrix} \quad (5)$$

Where "," indicate differentiation with respected to the symbol following the coma.

The derivatives in matrix [J] can be found from Eq. (1)

$$x_{,\xi} = \sum_{i=1}^{8} N_{i,\xi} x_i + \sum_{i=1}^{8} N_{i,\xi} \zeta \tfrac{h_i}{2} l_{3i}$$

$$x_{,\eta} = \sum_{i=1}^{8} N_{i,\eta} x_i + \sum_{i=1}^{8} N_{i,\eta} \zeta \tfrac{h_i}{2} l_{3i}$$

And so on

$$x_{,\zeta} = \sum_{i=1}^{8} N_i \tfrac{h_i}{2} l_{3i}$$

For this element, six types of non-zero strains exist, as follows:

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} u_{,x} \\ v_{,y} \\ w_{,z} \\ u_{,y} + v_{,x} \\ v_{,z} + w_{,y} \\ w_{,x} + u_{,z} \end{bmatrix} \quad (6)$$

The stress-resultant vector in the local coordinate system is,

$$\{N'\}=\{N_{x'}\ N_{y'}\ N_{x'y'}\ Q_{y'}\ Q_{x'}\ M_{x'}\ M_{y'}\ M_{x'y'}\}^T \quad (7)$$

Where :

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$Q_{y'}$ $Q_{x'}$: the shear stress per unit length in the x and y direction. The relationship between the stress resultants and the generalized strains can be stated as follows:

$$\{N'\}=[D']\{\varepsilon'\} \tag{8}$$

Where:

$[D']$: the rigidity matrix. A typical rigidity matrix for flat plate shell is given by:

$$[D'] = \frac{Eh}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{h^2}{12} & \frac{h^2\nu}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{h^2\nu}{12} & \frac{h^2}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h^2(1-\nu)}{24} \end{bmatrix} \tag{9}$$

k : shear shape factor (assumed k=1.2 for rectangular sections) (Hearn, 1985).

If the spacing of the stiffeners is uniform and they lie along the natural coordinate directions, equivalent shell rigidities can be obtained by merging the stiffener rigidities with those of the shell. Fig.3 shows a shell of thickness t with eccentric stiffeners in ξ and η directions at intervals s_ξ and s_η respectively (Hearn, 1985).The kinematics relations between the displacements at the rib centroidal axis and those at the shell midsurface are given in the coming Eq. (10), in which all the displacements are along the coordinate system x′,y′,z′ defined at the point under consideration, with z′ along the thickness of the shell, x′ tangential to the stiffener along the ξ direction, and y′ tangential to the stiffener along η direction.

For ξ rib: $w_{rx'} = w'$, $u_{rx'} = u' + e_\xi (\partial u'/\partial z')$
(10a)

For η rib: $w_{ry'} = w'$, $v_{ry'} = v' + e_\eta (\partial v'/\partial z')$
(10b)

Here u′, v′ and w′ are the displacements at the shell midsurface and e_ξ, e_η are the eccentricities of the stiffeners.

The stress resultants and strains of the ξ- direction rib are:

$$\left.\begin{aligned} \{N_{x'}\}_r &= 1/h\, \{N_{rx'}\ Q_{rx}\ M_{rx}\ T_{rx'}\}^T \\ \{\varepsilon_{x'}\}_r &= 1/h\, \{\varepsilon_{rx'}\ \gamma_{rx'z'}\ \chi_{rx'}\ \partial\theta_{rx'}/\partial x'\}. \end{aligned}\right\} \tag{11}$$

In which $N_{rx'}$,$Q_{rx'}$,$M_{rx'}$ and $T_{rx'}$ are the axial force, shear force, bending moment and tensional moment respectively. The same relation in η- direction rib exit. The relation between stiffener strains and shell strains are given by:

$$\begin{aligned} \varepsilon_{rx'} &= \varepsilon_{x'O} + e_\xi \chi_{x'} \\ \chi_{rx'} &= \chi_{x'} \\ \gamma_{rx'z'} &= \gamma_{z'x'O} \\ \partial\theta_{rx'}/\partial x' &= 1/2\ \chi_{x'y'} \end{aligned} \tag{12}$$

Similar expressions can be written for η- direction stiffeners. These relations for both sets of stiffeners may be expressed in matrix form as:

$$\{\varepsilon'\}_r = [T]\{\varepsilon'\} \tag{13}$$

Where:

[T] is a transformation matrix. The stress resultants in terms of strains can be written for both sets of stiffeners together as:

$$\{N'\}_r = [D']_r \{\varepsilon'\}_r \tag{14}$$

The various matrices in Eqs. (13) & (14) are given by:

$$\{\varepsilon'\}_r = \{\varepsilon_{rx'}\ \ \varepsilon_{ry'}\ \ \gamma_{rx'y'}\ \ \gamma_{ry'z'}\ \ \gamma_{rz'x'}\ \ \chi_{rx'}\ \ \chi_{ry'}\ \ \chi_{rx'y'}\}^T \tag{15}$$

$$\{N'\}_r = \{N_{rx'}\ N_{ry'}\ N_{rx'y'}\ Q_{ry'}\ Q_{rx'}\ M_{rx'}\ M_{ry'}\ M_{rx'y'}\}^T \tag{16}$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & e_\xi & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & e_\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{2} \end{bmatrix} \tag{17}$$

$$[D']_r = \begin{bmatrix} E_\xi A\xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_\eta A\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_\eta S_\eta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_\xi S_\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_\xi I_\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_\eta I_\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_\xi J_\xi + G_\eta J_\eta \end{bmatrix} \tag{18}$$

In Eq. (18), E, G, A, S, I and J respectively denote Young's modulus, shear modulus, cross sectional area, shear area, moment of inertia and torsional inertia of the ξ, or η- direction stiffener as indicated by the subscript. In Eq. (16), the quantity $M_{rx'y'}$ gives the sum of the torsional moments of both the stiffeners. If the stiffener rigidities are uniformly distributed over the spacing of the stiffeners to obtain equivalent rigidities over the shell midsurface, then from Eqs.(13) & (14), the strain energy of the stiffeners can be obtained as,

$$U_r = \frac{1}{2}\int \{\varepsilon'\}^T [T]^T [D]_r [T]\{\varepsilon'\}\ \ dA \tag{19}$$

in which $[D]_r$ is obtained by dividing the rigidity terms corresponding to ξ- ribs by s_ξ and those corresponding to η- ribs by s_η in $[D']_r$.

The total strain energy of the stiffened shell is then given by,

$$U = \frac{1}{2}\int \left(\{\varepsilon'\}^T [D']\{\varepsilon'\} + \{\varepsilon'\}^T [T]^T [D]_r [T]\{\varepsilon'\}\right) dA \tag{20}$$

This is equivalent to the behavior of a homogeneous shell with equivalent rigidity matrix $[D_{eq}]$ given by.

$$\left[D_{eq}\right] = [D'] + [T]^T [D]_r [T] \tag{21}$$

The stiffened shell then can be analyzed as a homogeneous shell using the element described earlier. To automate the stiffener spacing calculations, the following method can be implemented. Let n_ξ be the number of ξ- direction stiffeners and n_η is the number of η-direction stiffeners within the element. At a Gauss point, the following two values are computed:

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$$S_1 = 2(J_{21}^2 + J_{22}^2 + J_{23}^2)^{1/2}$$
$$S_2 = 2(J_{11}^2 + J_{12}^2 + J_{13}^2)^{1/2} \qquad (22)$$

in which J's are the coefficients of the Jacobian matrix. Effectively S_1 gives the dimension of the element along η-direction and S_2 gives the dimension along ξ- direction at that Gauss point. Thus,

$$\left.\begin{aligned} S_\xi &= \frac{S_1}{n_\xi} \\ S_\eta &= \frac{S_2}{n_\eta} \end{aligned}\right\} \qquad (23)$$

The kinetic energy T of structure is

$$T = \frac{1}{2}\{\dot{q}\}^T \int_V \rho\{N\}^T\{N\}dV\{\dot{q}\} \qquad (24)$$

Where:

$$[M] = \int_V \rho[N]^T[N] \; dV \qquad (25)$$

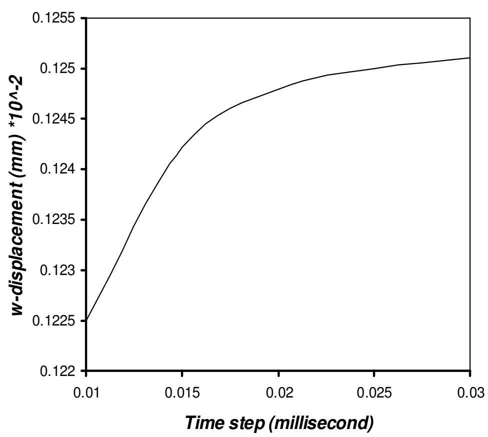

Fig.1: Variation of w-displacement with time step

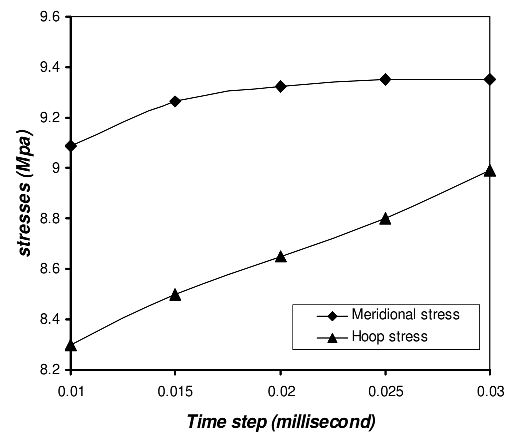

Fig.2: Variation of stresses ith time step

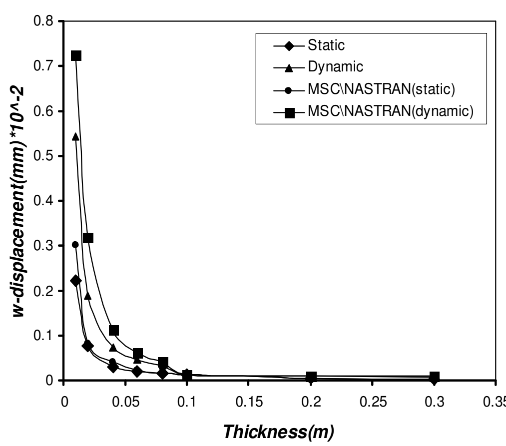

Fig.3: Variation of w-displacement with thickness

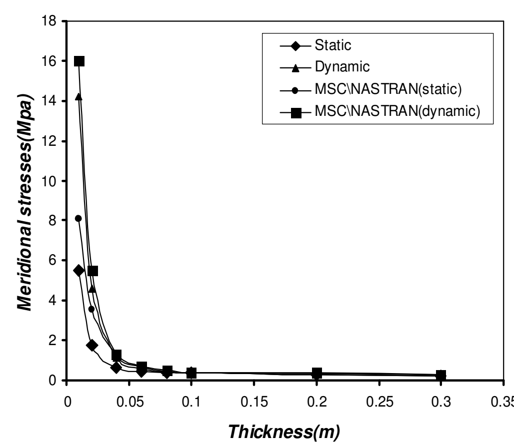

Fig.4: Variation of meridional stresses with thickness

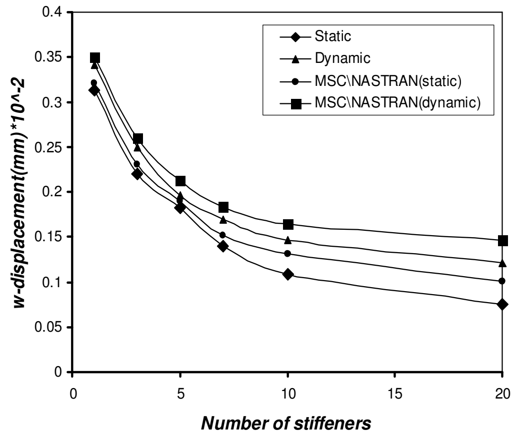

Fig.5: Variation of meridional stresses
number of stiffeners(string)

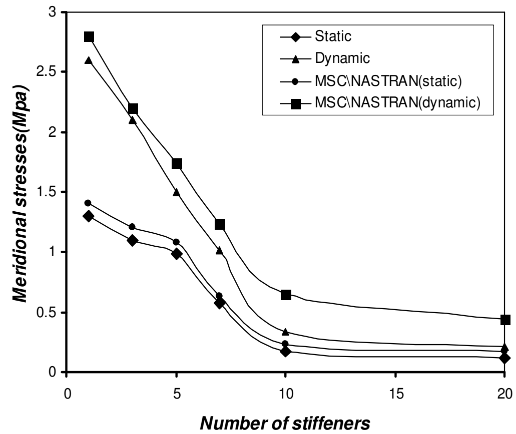

Fig.6: Variation of meridional stresses
number of stiffeners(string)

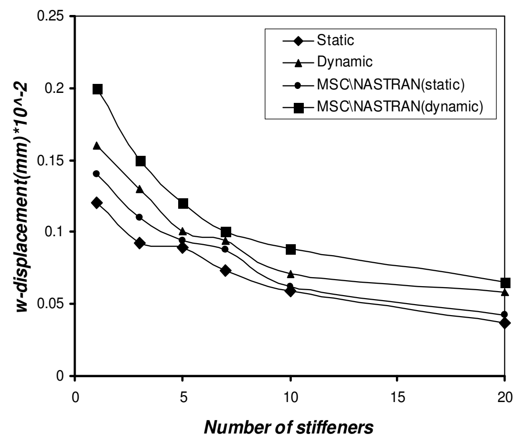

Fig.7: Variation of meridional stresses
number of stiffeners(ring)

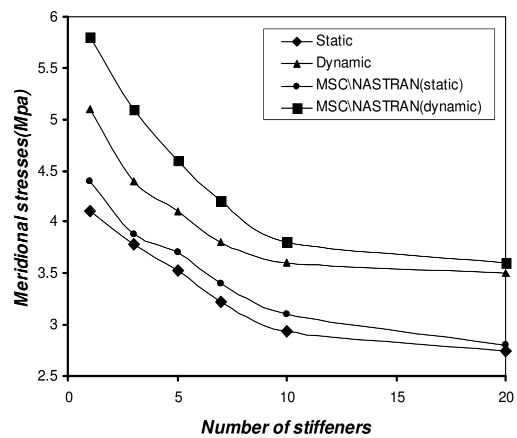

Fig.8: Variation of meridional stresses
number of stiffeners(ring)

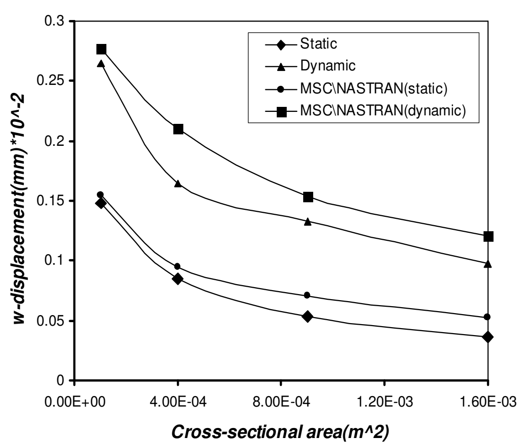

Fig.9: Variation of w-displacement
he cross-sectional area

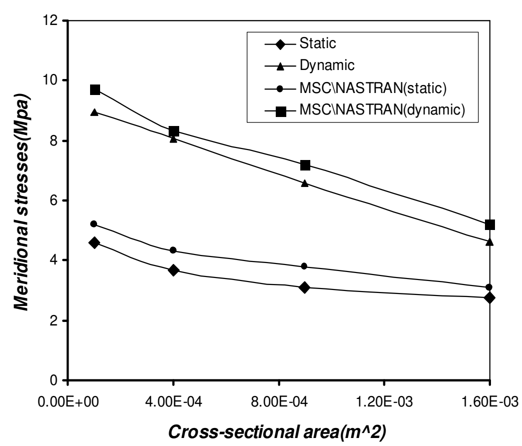

Fig.10: Variation of meridional stresses
the cross-sectional area

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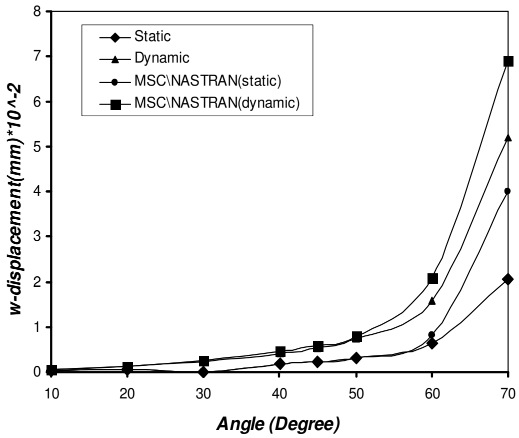

Fig.11: Variation of w-displacement th the angle of cone

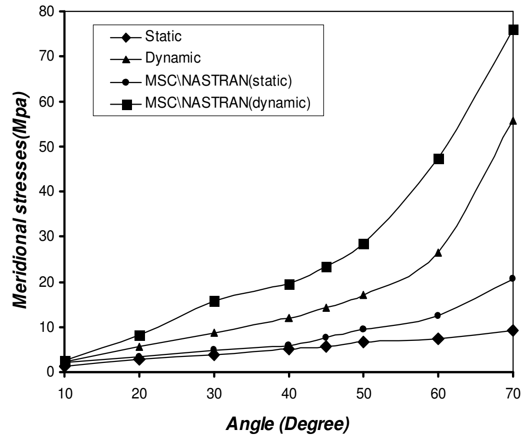

Fig.12: Variation of meridional stresses with the angle of cone

CONCLUSIONS

The conclusions obtained from the present analysis can be summarized as follows: the thickness conical shell is effect on the stresses and deformation. The stiffeners are reducing the strength-to-weight of the conical shell. The shape of the stiffeners (L-section) is greater effect on the bending and torsion resistance from the other section. Increment in the cross-sectional area of stiffener results reduces in the stresses and deformation. The rings have greater effect than the stringer on the conical shell, where the bending curvature is so much larger in the circumferential than the axial direction. The angle of the cone causes the higher stresses and larger deformation when it's reduced. It can be seen that the super parametric shell element gives a good results in such static analysis and dynamic analysis for stiffened conical shell. The results taken from the dynamic response is higher than static analysis.

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