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BELIEF IN CAUSATION:

ONE APPLICATION OF CARNAP'S INDUCTIVE LOGIC[1]

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ABSTRACT

This paper takes two tasks. The one is elaborating on the relationship of inductive logic with decision theory to which later Carnap planned to apply his system (§§1-7); this is a surveying side of this article. The other is revealing the property of our prediction of the future, subjectivity (§§8-11); this is its philosophical aspect. They are both discussed under the name of belief in causation. Belief in causation is a kind of "degree of belief" born about the causal effect of the action. As such, it admits of the analysis by inductive logic.

Keywords: Carnap's inductive logic, decision theory, belief in causation, subjectivity

§1 INTRODUCTION

When we intentionally act, we take future affairs into account. Suppose, e.g. one parks his car on some street; he will then wonder whether his parking will cause a traffic jam later. Let us call this mindset *belief in causation*. Sometimes the agent may care about a causal effect of his action in the future. It is the object of study below.

For its analysis, we dare to use Carnap's *inductive logic*. Some may think it miscasting, since Carnap's system is so formalized that it appears inapplicable to ordinary situations like the above. Nonetheless, in his later years, Carnap actually planned such an application (Carnap, 1962, p.vii). He noticed: the main field for his logic is *decision theory* rather than natural science; his logic takes on the pure, theoretical, logical part of *normative decision theory* (Carnap, 1971, p.26).

Thus, our investigation also directs itself toward decision theory, so that we shall deal with the relationship between inductive logic and decision theory in the first half of this paper, which includes the overview of decision theory (§2), the foundation of mathematical expectation (§3), and the actual application of inductive logic (§§5-7).

In the second half (§§8-11), the results of our investigation are examined; we shall particularly recognize *the subjectivity* of inductive logic.

§2 DECISION THEORY

In his later years, Carnap planned to apply his inductive logic to decision theory. But what on earth is the decision theory? To begin with, we must clarify it.

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[1] This paper was read at "Philosophy of Science Colloquium" in Institute Vienna Circle (the University of Vienna) on January 25, 2011. And this research was funded by Program for Evolving Humanities and Sociology (the University of Tokyo).

We may, for this purpose, refer to R. C. Jeffrey's book (Jeffrey, 1983)[2]. To introduce his explanation, let us imagine the following scenario:

(1) *An owner of a certain bookshop is wondering whether or not he should order a certain book, though its amount is only one. Reflecting on his past experience, the possibility of a customer coming to buy the book is 0.3. But once the book is sold, his gain will be 100 euro, where, of course, the charge for the ordering, 40 euro, is subtracted. Now, should he order the book?*[3]

According to Jeffrey, this scenario is decomposed into three factors: Act, Condition, and Consequence (Jeffrey, 1983, pp.1f.). *Act* is the option that the agent can choose: in (1), the agent is the owner, acts are "ordering the book" and "not ordering the book." *Condition* is the possible affair that has much to do with the acts: in (1), conditions are "a customer coming to buy the book" and "nobody coming to buy the book." *Consequence* is the possible affair that results from the chosen act together with a specific condition: in (1), consequences are "ordering the book and a customer coming to buy it," "ordering the book but nobody coming to buy it," "not ordering the book but a customer coming to buy one," and "not ordering the book and nobody coming to buy one." We can name these four consequences in terms of money, stating in order: "+100 euro," "−40 euro," "0 euro," and "0 euro." These are called *numerical desirability*.

These factors are put into the *desirability matrix* and the *probability matrix* respectively.

(2) *The Desirability Matrix for Situation (1)*

	A customer coming to buy the book	Nobody coming to buy the book
Ordering the book	+100 euro	−40 euro
Not ordering the book	0 euro	0 euro

(3) *The Probability Matrix for Situation (1)*

	A customer coming to buy the book	Nobody coming to buy the book
Ordering the book	0.3	0.7
Not ordering the book	0.3	0.7

By reference to these matrices, we can calculate *mathematical expectation*.

(4) *The mathematical expectation of ordering the book*
$= (+100) \times 0.3 + (-40) \times 0.7 = +2$ (euro)
(5) *The mathematical expectation of not ordering the book*
$= 0 \times 0.3 + 0 \times 0.7 = 0$ (euro)

These calculations are explained as follows: firstly, we multiply each correspondent entry between a desirability matrix and a probability matrix, and secondly, we add entries on the same row (because each row shows each act). And by their comparison, we can decide the best act; in the present case, since +2 > 0, the owner should order the book.

[2] Jeffrey was counted as Carnap's colleague (Carnap, 1962, p.xiii). Jeffrey also dedicated his book to Carnap (Jeffrey, 1983, p.xiv).
[3] This scenario is cited from Carnap, 1962, p.255, but a bit altered by Kaneko.

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§3 MATHEMATICAL EXPECTATION AS A WEIGHTED MEAN

This is an overview of decision theory, to which Carnap planned to apply his logic. But in the explanation above, it was not yet clarified how inductive logic works in that theory. So we must elucidate it further.

First, let us review the use of mathematical expectation, in terms of which optional acts were compared. But, on what ground? Calculations like (4) and (5) may possibly seem easy amalgams.

Carnap took this problem seriously; so he tried to ground mathematical expectation (Carnap, 1962, pp.521-522, pp.528-529). Take another look at (4) and (5). It may be noticed that they resemble an *average* or a *mean*: (4) resembles $\frac{(+100)+(-40)}{2} = +30$, and (5) resembles $\frac{0+0}{2} = 0$. From this viewpoint, Carnap attempted the justification of mathematical expectation (Carnap, 1962, pp.521f.). His idea is to ground it as a *weighted mean*.

The weighted mean is a well-known concept in the circles of investment, for example. Consider closing prices of a certain stock, supposing that it was 80 euro three months ago, 50 euro two months ago, and 20 euro last month. How can we calculate the *average* or the *mean* of this stock price? Presumably, at first, we use a *simple average*:

(6) $$\frac{80+50+20}{1+1+1} = 50$$

Yet, we are also in a position to evaluate *the latest* stock price, 20 euro, higher than *earlier* ones, 50 euro and 80 euro. Hence, we are led to differentiate each datum by *weighting* w_1 for 20, w_2 for 50, and w_3 for 80, where $w_1 > w_2 > w_3$. E.g. let $w_1=3$, $w_2=2$, and $w_3=1$. Then, the renewed average of the stock price is calculated as follows:

(7) $$\frac{80\times1 + 50\times2 + 20\times3}{1+2+3} = 40$$ [4]

We call this a *weighted mean*. Of course, it is calculated about other magnitudes than stock prices. In general, for magnitudes $m_1, \ldots, m_s$, their weighted mean is formulated as follows ("w_p" is a weight):

(8) $$\frac{\sum_{p=1}^{s}[m_p \times w_p]}{\sum_{p=1}^{s} w_p}$$ (Carnap, 1962, p.521)

Carnap's strategy was deriving mathematical expectation from this formula. While we have so far decided the weight (w_p) in terms of *time* (i.e. the *latest* datum is more highly weighted than *earlier* ones), Carnap took the *probability* of a hypothesis predicting a certain magnitude or a consequence to which the magnitude is assigned, instead. This idea is developed in the following way. Let h_p be such a hypothesis that predicts magnitude m_p itself or the consequence to which m_p is assigned. Then, the probability of h_p is symbolized as follows:

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[4] Why is the denominator of the left side not "1+1+1" but "1+2+3"? Of course, "1" corresponds to the weight of the datum "80", "2" to "50", and "3" to "20". The reason is known, transforming (7) into this:

(*) $$\frac{80 + (50 + 50) + (20 + 20 + 20)}{1+(1+1)+(1+1+1)}$$

This shows our way of evaluation in (7): we evaluate the latest datum, 20 euro, as worthy of *three* data, the earlier datum, 50 euro, as worthy of *two*, and the earliest, 80 euro, as worthy of only *one*, not changed. The numbers from 1 to 3 correspond to these evaluations.

(9) $c^*(h_p, e)$

Here we may already find a piece of inductive logic as well. The function "c*" is peculiar to inductive logic; it is called *c-function*, which assigns probability to each hypothesis "h_p" with regard to evidence "*e*" (Carnap. 1962, pp.293f.). Carnap's proposal was to replace the values of this function with the preceding time-values at w_p in (8). The result is as follows:

(10) $$\frac{\sum_{p=1}^{s}[m_p \times c^*(h_p,\ e)]}{\sum_{p=1}^{s} c^*(h_p,\ e)}$$

Carnap called this new formula *the probability₁-weighted mean* (Carnap, 1962, p.169).

With regard to its denominator, we obtain $\sum_{p=1}^{s} c^*(h_p,\ e) = 1$ from the well-known rule[5]. So it is eliminated, and the following holds:

(11) $\sum_{p=1}^{s}[m_p \times c^*(h_p,\ e)]$ (Carnap, 1962, p.522)

This is nothing but mathematical expectation[6].

§4 PROBLEM IN PROBABILITY MATRIX

In this way, Carnap justified the caluculation of mathematical expectation. His argument also tells us where inductive logic works; that is, "$c^*(h_p, e)$" in formula (11).

From this angle, let us review the scenario of a bookshop owner, adjusting (11) to its calculation, (4) and (5)[7]. As just stated, inductive logic is concerned with probability assignment. How about the case of the owner, then? On checking over his probability matrix (=3)[8], we know: he assigns probability *independently* of the chosen acts. The probability assignment to each condition ("a customer coming to buy the book," "nobody coming to buy the book") in each column in (3) are *not changed* whichever act ("ordering the book," "not ordering the book") might be chosen.

However, we often experience the opposite cases in real lives. As an example, let us take the case of *nuclear deterrence* (cf. Jeffrey, 1983, pp.2f., pp.8f.). Suppose a certain country deliberates on whether or not it should arm itself with a nuclear weapon; then, acts are "nuclear armament" and "nuclear disarmament," and conditions are "war" and "no war." In this situation, the country surely cannot think about the probability of each condition without taking chosen acts into account. In other words, it cannot think about the probability of war and of no war, independently of chosen acts. This is because the probability of war in the case of nuclear armament is obviously different from that in the case of nuclear disarmament. This is why the probability matrix in this case is formed in a different way from that of the bookshop owner (cf. Jeffrey, 1983, p.9); that is,

(12) *The Probability Matrix for Nuclear Armament*

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[5] Let sentences $h_1, \ldots, h_s$ be exhaustive and exclusive. Then the following holds:

(*) $\sum_{p=1}^{s} P(h_p) = 1$

Here "exhaustive" means "$h_1 \vee \ldots \vee h_s$ is logically true" and "exclusive" means "for any h_i, h_j ($1 \leq i, j \leq s$), $h_i \wedge h_j$ is logically false."

[6] Usually, instead of "$c^*(h_p, e)$," the notation "$P(h_p, e)$" is used. In more detail, see Kaneko, 2012b.

[7] The detail of this adjustment is as follows. "m_p" in (11) corresponds to "+100" and "−40" in (4), and to "0" and "0" in (5). "$c^*(h_p, e)$" in (11) corresponds to "0.3" and "0.7" in (4), and to "0.3" and "0.7" in (5).

[8] "m_p" in (11) corresponds to each entry of desirability matrix (2), and "$c^*(h_p, e)$" in (11) to each entry of probability matrix (3).

	War	No War
Nuclear Armament	0.1	0.9
Nuclear Disarmament	0.8	0.2

The influence of the acts on the conditions is clear in this matrix. Focus on, e.g. the lower left entry: if the country (agent) does nothing, in other words, chooses nuclear disarmament (act), then the probability of war (condition) becomes high (=0.8). On the contrary, see the upper left entry: if the country chooses nuclear armament, the probability of war gets lower (=0.1). In this way, the same condition has different probabilities depending on the chosen acts.

§5 BELIEF IN CAUSATION

This relationship between the act and the condition can be classified into *causation*[9]. So now, we have returned to our original interest, *belief in causation*. To discuss it further, then, let us take up the following scenario on the basis of the one mentioned at the beginning of this paper (§1)[10].

(13) *X is about to park his car on C street. But he does not want to cause a traffic jam. So he looks back to his past experience, and tries to predict the probability of a traffic jam in his parking. How does he make this prediction?*

We apply the preceding analyses of nuclear deterrence to this situation as well. It is, firstly, decomposed into parts: acts and conditions. Acts are "parking" and "not parking," conditions are "traffic jam" and "no traffic jam." We may put aside consequences or numerical desirability at present, because the pressing problem for X is not choosing an act but the probability of the conditions caused by his act. His interest is how probable the causal relationship between his parking and a later traffic jam is. We can then formulate it as follows:

(14) $c^*((\varepsilon \text{ causes a traffic jam}), e)$

Here, "ε" stands for an act[11]. "*e*" is the past data that X looks back to in situation (13). Like this, we think the chosen act (=ε) influences not the *probability* of a condition but the *condition itself* (=traffic jam), and assign probability to *causation as a whole*. This is the way of thinking developed in the following[12].

§6 APPLICATION OF INDUCTIVE LOGIC

Now then, let us scrutinize how we can apply inductive logic to our formulation (14).

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[9] Jeffrey also mentioned "causal influence" in decision theory (Jeffrey, 1983, p.24). However, his interest was rather in the problem of "evidential significance" found in Prisoner's Dilemma (Jeffrey, 1983, pp.15f.). Although he tried to deny those relations (Jeffrey, 1983, p.9), we may disregard those negative arguments. For Jeffrey merely made an extra convention to refuse them (ibid.).

[10] We might deal with the same situation, nuclear deterrence, here consecutively. However, for that, we need much more detailed preparations. So in this paper, we stick to the instance of parking.

[11] In this paper, I use "ε" as an individual constant for an event, and "e" as a variable for an event. In these notations, I follow Davidson's idea of *logic of events* (Davidson, 1980, Essay6). I ask the readers to make a distinction between "e" (=event) and "*e*" (= evidence).

[12] This treatment was presented initially in Kaneko, 2012a, where I elaborated on its technical matters as well. The relationship of inductive logic with causation was treated also in Uchii, 1972 and Uchii, 1974; but the main focus there was causal modality, not causation itself. Rather, as the work close to our argument, we may refer to Köhler, 2011. Again, as for so-called *probabilistic causality* (Salmon, 1980 and Suppes, 1970), I elaborated on why I do not favor their doctrines in Kaneko, 2012a, §2.

Review (13) first. We can name each event X has in mind "ε_1," "ε_2," "ε_3," and "ε_4." Therein, "ε_3" expresses that X parks his car on C street within a certain period of time; "ε_1" and "ε_2" are the past events similar to X's act, which X looks back to in (13); for example, "ε_1" expresses that Y parked his car on A street, and "ε_2" expresses that Z parked his car on B street; "ε_4" expresses something different from parking, not meaning a negative event like X's *not* parking; at present, we regard it as the future event that there is a rush of cars on C street. Again, "ε_1," "ε_2" and "ε_3" are arranged in the order of time, but we do not decide which precedes, "ε_3" or "ε_4," since we suppose that "ε_3" and "ε_4" have not been observed yet.

These "ε_1"~"ε_4" are regarded as individual constants. We put them in one class:

(15) Const. = $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$

This class is taken to be the expression of the *population* that X has in mind (cf. Carnap, 1962, p.207, pp.493f.). In the present situation, we divide it into two subclasses, further:

(16) $K_1 = \{\varepsilon_1, \varepsilon_2\}$
(17) $K_2 = \{\varepsilon_3, \varepsilon_4\}$

K_1 expresses events that X has already observed; we name it *the first sample*. K_2 expresses events that X has not observed yet or will not experience forever; we name it *the second sample*. The inductive inference we made hereafter is the inference from K_1 to one element in K_2. It is called the *singular predictive inference* (Carnap, 1962, p.207).

Next, we arrange two concepts applied to these individual constants: "parking a car" and "causing a traffic jam." As such, we introduce the following two *primitive monadic predicates*.

(18) Pred. = {_is parking, _causes a traffic jam}

From (17) and (18), we can form an artificial language "$\mathfrak{L}_4^2$"[13]. $\mathfrak{L}_4^2$ contains the logical symbols of first-order predicate logic as well.

Using $\mathfrak{L}_4^2$, we can compile a probability matrix for situation (13).

(19) *The Probability Matrix for Situation (13)*

	Traffic Jam	No Traffic Jam
Parking	c* ((ε_3 causes a traffic jam), {(ε_1 is parking)∧(ε_1 causes a traffic jam)} ∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)} ∧(ε_3 is parking))	c* (¬(ε_3 causes a traffic jam), {(ε_1 is parking)∧(ε_1 causes a traffic jam)} ∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)} ∧(ε_3 is parking))
Not Parking	c* ((ε_4 causes a traffic jam), {(ε_1 is parking)∧(ε_1 causes a traffic jam)} ∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)} ∧ ¬(ε_4 is parking))	c* (¬(ε_4 causes a traffic jam), {(ε_1 is parking)∧(ε_1 causes a traffic jam)} ∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)} ∧ ¬(ε_4 is parking))

Here, see, e.g. the left upper entry:

(20) c* ((ε_3 causes a traffic jam), {(ε_1 is parking)∧(ε_1 causes a traffic jam)}∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)}∧(ε_3 is parking))

This means: "Y's parking on A street (=ε_1) caused a traffic jam, Z's parking on B street (=ε_2) caused a traffic jam… Therefore, If I (=X) parks my car on C street, then it (=ε_3) will cause a traffic jam." This is quite similar to *inductive inference* like "This swan is white, that swan is white… Therefore, the next swan will be white." In this way, we can treat X's belief in causation as a kind of inductive inference in inductive logic.

[13] In general, the language of inductive logic is symbolized as $\mathfrak{L}_N^\pi$, where N expresses the number of individual constants, and π expresses the number of predicates (Carnap, 1962, p.123). As for the relationship between artificial languages and inductive logic, see Kaneko, 2012b.

§7 Probability Assignment

This is the formulation of belief in causation in inductive logic. How, then, can we assign a concrete values to it? Let us pursue this issue further.

In the first place, turn our eyes to the parallelism between (14) and (9), in terms of which "(ε_3 causes a traffic jam)" in (20) is taken to be a *hypothesis* "h_p" in (9), and "{(ε_1 is parking a car)∧(ε_1 causes a traffic jam)}∧...∧(ε_3 is parking a car)" in (20) to *evidence* "*e*" in (9).

At present, it suffices to take the hypothesis merely as an expression of the causal relationship[14]. The more confusing here is the form of evidence:

(21) {(ε_1 is parking)∧(ε_1 causes a traffic jam)}∧{(ε_2 is parking)∧(ε_2 causes a traffic jam)}∧(ε_3 is parking)

What does this long sentence mean? Let us, then, start our application of inductive logic with the explanation of this formula.

(22) The conjunction stating, over s individual constants and p molecular predicates forming a division, in the following way, which predicate is predicated of which individual constant is called an *individual distribution*.

$$e_k = \ulcorner M_{k1}(\varepsilon_{j1}) \wedge M_{k2}(\varepsilon_{j2}) \wedge ... \wedge M_{ks}(\varepsilon_{js}) \urcorner$$ (Carnap, 1962, p.111)[15]

"M_{k1}" to "M_{ks}" is one of p molecular predicates. "{(ε_1 is parking)∧(ε_1 causes a traffic jam)}" and "{(ε_2 is parking)∧(ε_2 causes a traffic jam)}" in (22) are regarded as such molecular predicates. What is "molecular predicate," then?

(23) Only for abbreviation, we write, e.g. "$P_1 \wedge P_2(e_1)$" instead of "$P_1(e_1) \wedge P_2(e_1)$" and we call such an expression a *molecular predicate expression*. Moreover, we write, e.g. "$M(e_1)$" instead of "$P_1 \wedge P_2(e_1)$" and call such an expression a *molecular predicate* (Carnap, 1962, pp.104-105).

Here, "P_1" and "P_2" are primitive monadic predicates; e.g. "_is parking" and "_causes a traffic jam" in $\mathfrak{L}_4^2$ (cf. (16)). Of the possible molecular predicates in $\mathfrak{L}_4^2$, the following four are the most important:

(24) $\forall e[Q_1(e) \leftarrow\rightarrow (e \text{ is parking}) \wedge (e \text{ causes a traffic jam})]$, $\forall e[Q_2(e) \leftarrow\rightarrow (e \text{ is parking}) \wedge \neg(e \text{ causes a traffic jam})]$
$\forall e[Q_3(e) \leftarrow\rightarrow \neg(e \text{ is parking}) \wedge (e \text{ causes a traffic jam})]$, $\forall e[Q_4(e) \leftarrow\rightarrow \neg(e \text{ is parking}) \wedge \neg(e \text{ causes a traffic jam})]$

"Q_1" to "Q_4" are called *Q-predicates*. Their formal definition is as follows:

(25) The molecular predicates introduced with the following definition are called *Q-predicates*.

$$\forall e[Q_i(e) \leftarrow\rightarrow (\neg)P_1(e) \wedge ... \wedge (\neg)P_\pi(e)]$$ (Carnap, 1962, p.125)

In (22), it was not clear what "division" is, either; so we must follow it up:

(26) If molecular predicates $M_1,\dots,M_p$ fulfill the following conditions, then they are called *forming a division*.
① $\vDash$[16] $\forall e[M_1(e) \vee ... \vee M_p(e)]$ (exhaustiveness)
② For any M_i, M_j $(1 \leq i,j \leq p)$, $\vDash \forall e \neg [M_i(e) \wedge M_j(e)]$ (exclusiveness)
③ For no M_i $(1 \leq i \leq p)$, $\vDash \neg \exists e M_i(e)$ (M_i is not logically empty) (Carnap, 1962, pp.107-108)

Now we realized the preceding definition (=22); and by its aid, (21) is reformulated into "$Q_1(\varepsilon_1) \wedge Q_1(\varepsilon_2) \wedge$($\varepsilon_3$ is parking)." But here, an atomic sentence "(ε_3 is parking)" still remains. How should we deal with it?

(27) Any formula (e) in $\mathfrak{L}_N^\pi$ is expressed with a disjunction of Q-predicates in the following way;

$$\forall e[\mathfrak{M}(e) \leftarrow\rightarrow Q_{i1}(e) \vee Q_{i2}(e) \vee ... \vee Q_{iw}(e)]$$ (Carnap, 1962, pp.107-108)[17]

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[14] In Kaneko, 2012a, I have fully dealt with the problems of expressing the causal relationship in inductive logic.
[15] "⌜ ⌝" is Quine's quasi-quotes. But I place legibility prior to strictness. The undefined concepts "molecular predicates" and "division" are explained below. As for molecular predicates, see (24); as for a division, see (27).
[16] "⊨" means "logically true" though Carnap wrote "⊢" (Carnap, 1962, p.83).
[17] I omit its proof. See Kaneko, 2010b, (18).

By this theorem, "(ε_3 is parking)" is replaced with "$Q_1(\varepsilon_3) \vee Q_2(\varepsilon_3)$." The number of Q-predicates we can substitute for each sentence is called its *width* (Carnap, 1962, p.127). It is marked with the second subscript "w" of the last disjunct in (27). The width of "(ε_3 is parking)" is two. The width of "{(e is parking a car)∧(e causes a traffic jam)}" is one.

On the basis of this terminology, the formula playing a central role in our argument is introduced:

(28) Let "s_M" be the number of individual constants which molecular predicate M is predicated of in individual distribute *e* as evidence. And let "w_M" be the width of M, and "s" the number of the first sample, i.e. the number of observed individual constants. Then, the probability that M is predicated of the next individual constant "ε_{s+1}"—a member of the second sample—is calculated with the following formula:

$$\mathfrak{c}^*(M(\varepsilon_{s+1}), e) = \frac{s_M + w_M}{s + \kappa} \qquad \text{(Carnap. 1962, p.568)}$$ [18]

By means of this, we can assign concrete values to each entry of probability matrix (20):

(29) $\mathfrak{c}^*((\varepsilon_3 \text{ causes a traffic jam}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge (\varepsilon_3 \text{ is parking}))$

$$= \frac{\mathfrak{c}^*((\varepsilon_3 \text{ is parking}) \wedge (\varepsilon_3 \text{ causes a traffic jam}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\})}{\mathfrak{c}^*((\varepsilon_3 \text{ is parking}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\})}$$ [19]

$$= \frac{\mathfrak{c}^*(Q_1(\varepsilon_3), Q_1(\varepsilon_1) \wedge Q_1(\varepsilon_2))}{\mathfrak{c}^*(Q_1(\varepsilon_3) \vee Q_2(\varepsilon_3), Q_1(\varepsilon_1) \wedge Q_1(\varepsilon_2))} \quad \text{from (27) and (24)}$$

$$= \frac{\frac{2+1}{2+4}}{\frac{2+2}{2+4}} \quad \text{from (28)}$$

$$= \frac{3}{4}$$

This value $\frac{3}{4}$ occupies the left upper entry in matrix (19). Likewise,

(30) $\mathfrak{c}^*(\neg(\varepsilon_3 \text{ causes a traffic jam}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge (\varepsilon_3 \text{ is parking})) = \frac{1}{4}$

(31) $\mathfrak{c}^*((\varepsilon_4 \text{ causes a traffic jam}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge \neg(\varepsilon_4 \text{ is parking})) = \frac{1}{2}$

(32) $\mathfrak{c}^*(\neg(\varepsilon_4 \text{ causes a traffic jam}), \{(\varepsilon_1 \text{ is parking}) \wedge (\varepsilon_1 \text{ causes a traffic jam})\} \wedge \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge \neg(\varepsilon_4 \text{ is parking})) = \frac{1}{2}$

With these values, probability matrix (19) is rewritten as follows:

(33) *The Revised Version of (19)*

	Traffic Jam	No Traffic Jam
Parking	$\frac{3}{4}$	$\frac{1}{4}$
Not Parking	$\frac{1}{2}$	$\frac{1}{2}$

§8 SUBJECTIVITY OF INDUCTIVE LOGIC

This is the overview of our application of inductive logic for the analysis of belief in causation. Given more information, we can compile a desirability matrix, calculate the mathematical expectation, and find the act to be chosen. But we omit this process. The more interesting here is that the language $\mathfrak{L}_4^2$ was *designed*, and that the design was made from the agent's personal viewpoint (see (16), (17), (18) and (19) above). Let us take up this property, calling it *the subjectivity of inductive logic*.

[18] This formula is located at the climax of inductive logic though I omit its proof (see Kaneko, 2010b, §§16-19).
[19] From the following theorem:

(*) $\mathfrak{c}(h, e \wedge i) = \frac{\mathfrak{c}(h \wedge i, e)}{\mathfrak{c}(i, e)}$ (Carnap, 1962, p.317 T59-1.n.(2))

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As for the *design* of language, its importance was fully realized by Carnap (Carnap, 1962, p.54). But he would have denied its *subjectivity*. For, as we see in his earlier works, his focus was exclusively on natural sciences, in which everything must be made *objectively*.

Nevertheless, in his later works, even Carnap came to recognize the subjectivity (cf. §1); we cannot but make inductive inference *subjectively*. Review the situation (13): there is no telling whether such and such parking causes a traffic jam on such and such a street, since we do not know any general laws in that situation[20]. All we can do is look back to the past experience, as X did—but the past experience is different from person to person, which is why our inference must be *subjective*.

This is how the subjectivity steps into our picture. We may, in a formal way, ascribe it to the personal design of language. Let me describe it below.

X was in a position to design *another* language. For example, suppose X removed "ε_1" from his universe of discourse (=15). Where "ε_1" means that Y parked his car on A street, X may say, "Y is different from me. He is much more careful than me. And the traffic of A street is different from that of C street. So I cannot include Y's instance." Then, the language is changed to $\mathfrak{L}_3^2$ designed from three individual constants, i.e. {ε_2, ε_3, ε_4} and the same predicates, i.e. {_is parking, _causes a traffic jam}[21]. In $\mathfrak{L}_3^2$, the result is as follows:

(34) $c^*((\varepsilon_3 \text{ causes a traffic jam}), \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge (\varepsilon_3 \text{ is parking})) = \frac{2}{3}$

(35) $c^*(\neg(\varepsilon_3 \text{ causes a traffic jam}), \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge (\varepsilon_3 \text{ is parking})) = \frac{1}{3}$

(36) $c^*((\varepsilon_4 \text{ causes a traffic jam}), \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge \neg(\varepsilon_3 \text{ is parking})) = \frac{1}{2}$

(37) $c^*(\neg(\varepsilon_4 \text{ causes a traffic jam}), \{(\varepsilon_2 \text{ is parking}) \wedge (\varepsilon_2 \text{ causes a traffic jam})\} \wedge \neg(\varepsilon_3 \text{ is parking})) = \frac{1}{2}$

So the probability matrix for situation (14) becomes as follows:

(39) *The Probability Matrix for another language of X's*

	Traffic Jam	No Traffic Jam
Parking	$\frac{2}{3}$	$\frac{1}{3}$
Not Parking	$\frac{1}{2}$	$\frac{1}{2}$

Compare this table with (34) above. Obviously, the probability of upper entries is changed. In this way, the agent's design of language directly influences his reasoning.

§9 AS THE MOST RADICAL THEORY OF SUBJECTIVITY

This is how we can ascribe the subjectivity to the agent's personal design of language. Carnap might have rejected this subjectivity; yet, taking the preceding analyses into account, even he had to admit it. Let us ascertain this point further in comparison with the tenet seemingly more relevant to our present idea: Ramsey's *subjective theory*[22]**.**

[20] Even statistical laws are no more than additional evidence. The crucial point here is that the situation is completely personal. As for this, see also Kaneko, 2012a.

[21] X could also change the design of predicates, too. For example, he could categorize his act more widely as a stop. In this case, the predicate "_is parking" is replaced with "_is a stop." And the number of individual constants would increase since the number of events belonging to a stop is larger than that to parking.

[22] I have fully dealt with Ramsey's theory in Kaneko, 2007.

Ramsey's *subjective theory* is a standpoint that identifies probability with *a degree of belief* (Ramsey, 1926). A degree of belief is, according to him, measured by a *bet*. Imagine we offer a certain bet to an agent, and he shows an *indifferent* attitude. Then, the *betting rate* of this bet is regarded as a *fair* betting quotient for him, and identified with his degree of belief in the *condition*[23].

Subjective theory does not impose any limitations on this process: the agent can initially bear his degree of belief as he likes[24]. In this sense, the probability is literally *subjective*.

Does Carnap's theory have this property as well? My answer is yes; besides, in my opinion, it shows that property more radially than Ramsey. For Ramsey's theory does not explain why the agent bears such and such a degree of belief. It is true that it lets the agent freely bear any degree of belief; but it does not tell us why he bears *such a degree*. In this very respect, inductive logic supersedes the subjective theory, because, as we saw above (§8), inductive logic completely explains why the agents bears such a degree of belief in terms of the design of language.

§10 RELATIONSHIP BETWEEN SUBJECTIVE THEORY AND INDUCTIVE LOGIC

That is why inductive logic is considered to be more "subjective" than Ramsey's theory. But here remains one more, final obstacle to be removed. That is, Carnap's standpoint was *logical theory*, which is to be strictly distinguished from Ramsey's theory[25].

Logical theory was originally advanced by Keynes and Harold Jeffreys. According to it, probability is a *partial logical implication* from evidence to a hypothesis. Carnap properly merged his theory into this standpoint[26].

In spite of this discrepancy, we may find a clue to reconcile logical theory with subjective theory. In his later works, Carnap suddenly stopped his refusal of subjectivity. This is, on the one hand, because he could not help admitting the phenomenal rise of the subjective theory (Carnap, 1962, pp.xivf; Carnap, 1971, pp.13f.), but on the other hand, because he was sure that the subjective theory is reducible to his logical theory. It is this latter stance that we take as a clue.

According to Carnap, the subjective theory is no more than *qualified psychologism*. We can understand "psychologism" here with the distinction of *subjectivism* and *objectivism*[27], which are names given to the standpoints in deductive logic (Carnap, 1962, pp.37-42).

Subjectivism regards logical formulas as mental laws; thus, it is called *psychologism* as well. This standpoint was advanced and shared among logicians so vaguely[28]. But they soon modified their tenet, making a distinction between *descriptive* and *normative*. And in terms of

[23] In detail, see Ramsey, 1926, pp.68f., Carnap, 1962, pp.165f., or Jeffrey, 1983, pp.41f.

[24] After that, however, different beliefs will converge into one degree by the aid of Bayes's theorem as relevant evidence is supplemented. This is the strategy of subjective theory.

[25] See Carnap, 1962, pp.xiv-xv, pp.42-51, pp.162-175, and Carnap, 1966, pp.19-39.

[26] In Kaneko, 2012b, I fully elaborated on this "logical" feature .

[27] Let me get through with the explanation of objectivism here. This standpoint was advanced by Frege and Husserl. According to it, logical formulas are regarded as the expression of objective relationship among propositions. It is obvious that this standpoint is a predecessor of Carnap's logical theory.

[28] As an example, Carnap barely took the introduction of Bool's masterpiece, *Laws of thought* (Carnap, 1962, p.40).

the latter, i.e. *normative subjectivism*, logical formulas were considered to be the *norms* for rational thinking (Carnap, 1962, pp.41f.).

It is this standpoint that Carnap called "qualified psychologism," and he identified it with Ramsey's subjective theory in the context of philosophy of probability. This is because Ramsey's famous *Dutch book argument* (Ramsey, 1926, p.78) showed much the same view as normative subjectivism. According to that argument, the agent not following the laws of probability will be deceived by a cunning better. This shows that Ramsey regarded formulas as norms.

On normative subjectivism or qualified psychologism, Carnap commented that it added nothing to objectivism (Carnap, 1962, p.41, p.47)[29]. That is why Carnap assimilated Ramsey's subjective theory (regarded as normative subjectivism or qualified psychologism) with his logical theory[30].

§11 CONCLUSION

Now we realized how deeply the subjectivity underlies human understanding of probability or inductive inference. This insight was also gained by Laplace, a founder of probability theory.

(40) *Even if the same information is given to them, different agents might have different degrees of belief according to their extent of knowledge* (Laplace, 1825, p.10).

Laplace here emphasized the relationship of probability theory and *human ignorance* (Laplace, 1825, p.7). This ignorance gives rise to the discrepancy among human degrees of belief, which leads to our motif: subjectivity. Ironically, Laplace's ground rule, *the principle of indifference,* was also criticized as such (cf. Uchii, 1995, pp.192-195).

The subjectivity inevitably rules our thought of probability. We concluded from this endpoint that our inductive inference, which has a kinship with probability theory, is also subjective. You may fear the future, and try to predict it; but such prediction is desperately subjective.

So far we have attributed this feature especially to our belief in causation. For some readers, this conclusion appears negative: our prediction of the future is neither objective nor reliable. But what I wish to insist is the opposite: our future is open, because nobody forecasts it beforehand.

[29] Let me explain this point in more detail here. In the first place, the reason why logicians left "primitive psychologism" (Carnap, 1962, p.47) was that they did not wish to think of their study as the study of mental laws (Carnap, 1962, p.41). Thus, they began explaining, e.g. logical consequence ($i \vDash j$) in terms of normativity as follows:

(i) If somebody has sufficient reasons to believe in the premise *i*, then the same reasons justify likewise his belief in the conclusion *j*. (Carnap, 1962, p.41)

However, Carnap criticized this view for adding nothing to the following simple explanation of logical theory;

(ii) If *i* is true, then *j* is necessarily also true. (ibid.)

Therefore, it was said that qualified psychologism adds nothing to objectivism.

[30] Carnap also referred to the passages where Ramsey characterized probabilistic theory as logic (Ramsey, 1926, p.82 etc.)

REFERENCES (Translations and alternations of the texts are often made without notification)

Carnap,R. (1962). *The Logical Foundations of Probability* 2nd ed. The University of Chicago Press.

Carnap, R. (1966). *Philosophical Foundations of Physics.* Basic Books.

Carnap, R. (1971). Inductive Logic and Rational Decisions. *Studies in Inductive Logic and Probability* Vol. I. University of California Press.

Jeffrey, R.C. (1983). *The Logic of Decision* 2nd ed., The University of Chicago Press.

Kaneko, Y. (2007). Belief as Probability. *Philosophical Studies XXV*. The University of Tokyo.(Japanese)

Kaneko, Y. (2009). *The Phases of Ethical Judgements: From a Motivistic Point of View*. A doctoral dissertation submitted to the University of Tokyo. (Japanese)

Kaneko, Y. (2012a). The Confirmation of Singular Causal Statement by Carnap's Inductive Logic. *The Logica Yearbook 2011*.

Kaneko, Y. (2012b). Carnap's Thought on Inductive Logic. *Philosophy Study*, *10th issue*.

Laplace, P. (1825). *Essai Philosophique sur les Probabilités 5e ed*, Cambridge U.P.

Davidson,D. (1980). *Essays on Actions and Events*, Oxford University Press.

Ramsey, F.P. (1926). Truth and Probability. *Philosophical Papers: F.P.Ramsey*, Oxford U.P.

Köhler, E. (2011). Unification of Probability Concepts. (Unpublished)

Salmon, W. (1980). Probabilistic Causality. *Pacific Philosophical Quartely*, *61*.

Suppes, P. (1970). *A Probabilistic Theory of Causality*. North-Holland Publishing Company.

Uchii, S. (1995). *An Introduction to Philosophy of Science*, Sekaishisousya.

Uchii, S. (1972). Inductive Logic with Causal Modalities: A Probabilistic Approach. *Philosophy of Science* **39**.

Uchii, S. (1974). Inductive Logic with Causal Modalities: A Deterministic Approach. *Synthese* **26**.