VISCOUS DISSIPATION EFFECT ON THE MIXED CONVECTION MHD FLOW TOWARDS A STAGNATION POINT WITH CONVECTIVE BOUNDARY CONDITION IN A POROUS MEDIA

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ABSTRACT

An analysis of the viscous dissipation effect of the mixed convection MHD flow towards a stagnation point with convective boundary condition embedded in a porous media has been the subject of this research work. The nonlinear partial differential equations arising from the flow modeling were transformed into coupled nonlinear ordinary differential equations and subsequently solved using the Runge-Kutta shooting method. Effect of the relevant thermo-physical parameters has also been numerically investigated and visualized.

Keywords: viscous dissipation, convective boundary, stagnation point, MHD.

INTRODUCTION

Stagnation point flow has become an interesting area of research due to its varied applications both in industrial and scientific applications such as extrusion of polymers, cooling of metallic plates, aerodynamics plastic extrusion, glass blowing and fiber spinning etc. Since the work of Hiemenz[1] who studied the two dimensional flow of an incompressible viscous fluid, several works has been published in literature. For example Ramachandran et'al[6] studied the stagnation point flow towards a heated vertical plate, he considered both the arbitrary wall temperature and arbitrary surface heat flux. Ishak et'al[2] also studied the stagnation point flow but considered the case of a porous wall. Okedayo et'al[5] also carried out the analysis of plane stagnation point flow with convective boundary conditions.

Heat transfer characteristics in stagnation point flow has also been studied by several authors, for example Attia[13] studied the MHD stagnation point flow with heat transfer over a permeable surface, Massoudi and Ramezan[8] carried out an analysis of the boundary layers heat transfer of a viscoelastic fluid at a stagnation point. While Chiam[9] studied the heat transfer with variable conductivity in a stagnation point flow towards a stretching sheet.

on the MHD stagnation point flow, several works has been published which include the papers of Ariel[11], Chamkha[10] and Attia[12]

On the flow with convective boundary conditions, Aziz[14] studied a similarity solution applied to laminar thermal boundary layer flow over a flat plate with a convective surface boundary conditions.

Makinde and Olanrewaju[7] also presented a study on the buoyancy effects on thermal boundary layer over a flat vertical plate with a convective surface boundary condition. Okedayo et'al[15] presented the effects of viscous dissipation on the mixed convection heat transfer over a vertical plate with internal heat generation and convective boundary conditions. But the problem of viscous dissipation effect on the mixed convection MHD flow towards a stagnation point with convective boundary condition in a porous media has been neglected which is the motivation of this present work.

Hence the focus of this study is to investigate the effect of viscous dissipation and convective boundary condition on MHD flow towards a stagnation point.

MATHEMATICAL FORMULATIONS

A two-dimensional body is placed in a stream of quiescent fluid. We consider heat transfer near the upstream stagnation line, where the flow is assumed to be laminar. The problem is restricted to the case of a plane plate perpendicular to the stream. It is assumed that the fluid properties are constant except for the fluid viscosity which vary as an inverse linear function of temperature as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\frac{\partial^2 u}{\partial y^2}+S(x)g\beta(T-T_\infty)-\frac{\sigma B^2 u}{\rho}-\frac{\gamma}{k}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\gamma}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

subject to

$$\left.\begin{array}{l} u=0, v=0, -k\dfrac{\partial T}{\partial y}(x,0)=h_f\left(T_f-T(x,0)\right), at\ y=0 \\ u=U(x), T\to T_\infty, as\ y\to\infty \end{array}\right\} \qquad (4)$$

Where u is the velocity, T is the temperature, T_W is the wall temperature, $\mathrm{U(x)=U_0}x$ is the fluid free stream velocity, p is the pressure, B is the magnetic field, γ is the kinematic viscosity, C_p is the specific heat capacity, ρ is the fluid density, σ is the electrical conductivity, α is the thermal diffusivity and (x, y) are the coordinate axes, S(x)=x is the body force. In order to solve (1)-(4) we introduce the following similarity variables,

$$\left.\begin{array}{l} \eta\eta=y\sqrt{\dfrac{U(x)}{\gamma x}}, \psi(x,y)=\sqrt{\gamma x U(x)}, u=\dfrac{\partial\psi}{\partial y}, v=-\dfrac{\partial\psi}{\partial x}, \Pr=\dfrac{\gamma}{\alpha} \\ M=\dfrac{\sigma B_0}{\rho U_0}, \theta(\eta)=\dfrac{T-T_\infty}{T_W-T_\infty}, Ec=\dfrac{U^2x^2}{C_p(T_W-T_\infty)} \end{array}\right\} \qquad (5)$$

Substituting the above variables in equations (1)-(4), we obtain

$$f''' + ff'' - f'^2 + G\theta - (\frac{1}{Da} + H)(f' - 1) + 1 = 0 \quad (6)$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Ecf''^2 = 0 \quad (7)$$

$$f'(0) = f(0) = 0, \theta'(0) = -Bi(1 - \theta(0)), f'(\infty) = 1, \theta(\infty) = 0 \quad (8)$$

where G is the Grasshof number, Pr is the Prandtl number, H is the magnetic parameter, Da is the Darcy number, Ec is the Eckert number and Bi is the Biot number. and prime denote differentiation with respect to η.

NUMERICAL RESULTS AND DISCUSSION

The nonlinear equations (6)-(8) are solved numerically using the classical fourth order Runge-Kutta method together with the shooting technique implemented on a computer program written in Maple (14). A convenient step size was chosen to obtain the desired accuracy.

Fig.1 represents the velocity profile for various values of the Grasshof number while Pr, Da, H, Ec, and Bi are kept constant at 0.71, 0.1, 1, 0.5 and 0.2 respectively. it is observed that increase in the Grasshof number leads to a corresponding increase in the velocity profile, a similar trend is observed for the temperature profile in Fig.4.

Fig.2 and Fig.5 shows the velocity and temperature profile for various values of the Darcy number while G, Pr, Da, H, Ec, and Bi are kept constant at 0.1, 0.71, 1, 0.5 and 0.2 respectively, it seen that increase in the Darcy number leads to a decrease in the velocity and temperature profiles.

In Fig.3 and Fig.7 we depicts the velocity and temperature profiles for various values of the magnetic parameter with Pr, Da, G, Ec, and Bi kept at 0.71, 0.1, 0.1, 1, 0.5 and 0.2 respectively. it is clearly seen that increase in the magnetic parameter leads to an increase in the profiles of velocity and temperature respectively.

Fig.6 shows the temperature profile for various Prandtl number, it is seen that the profiles increases for various values of the prandtl number near the wall while the converse is seen far away from the boundary later into the free stream.

While in Fig.8 we see the temperature profile for various values of the Eckert number, it is seen that increase in the Eckert number leads to increase in the temperature. and Fig.9 shows that the temperature profile increases for increasing values of the Biot number.

In Table.1 we show the values of the skin friction coefficient and the Nusselt numbers for values of the necessary thermo-physical parameters.

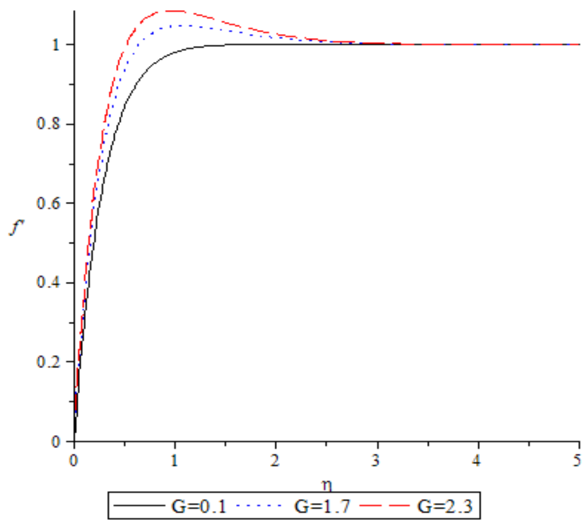

Figure 1: Velocity profile for various values of the Grasshof number for Pr=0.71, Da=0.1, H=1, Ec=0.5, Bi=0.2

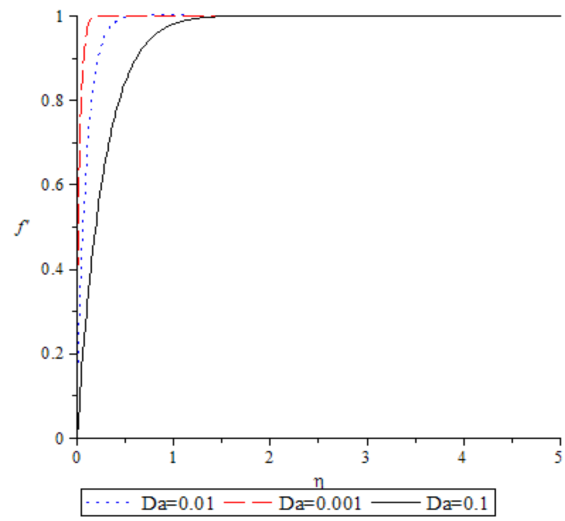

Figure 2: Velocity profile for various values of the Darcy number for Pr=0.71, G=0.1, H=1, Ec=0.5, Bi=0.2

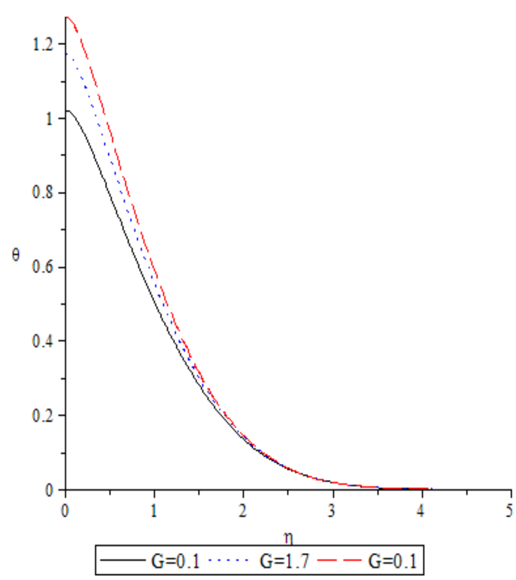

Figure 4: Temperature profile for various values of the Grasshof number for Pr=0.71, Da=0.1, H=1, Ec=0.5, Bi=0.2

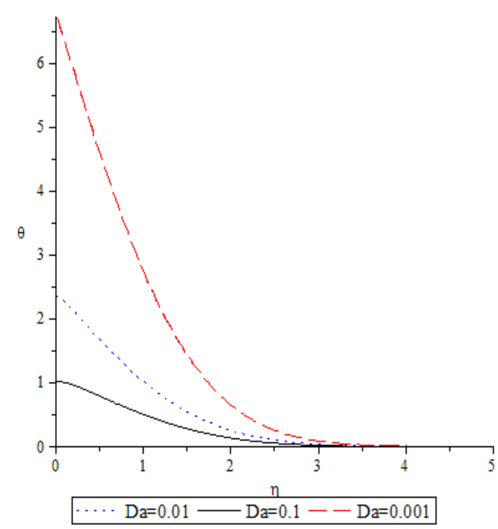

Figure 5: temperature profile for various values of the Darcy number for Pr=0.71, G=0.1, H=1, Ec=0.5, Bi=0.2

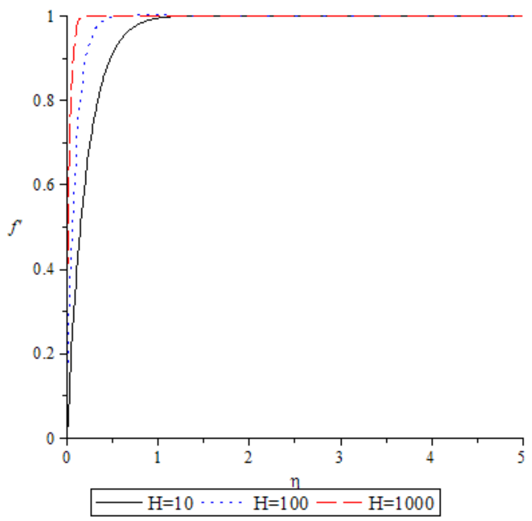

Figure 3: Velocity profile for various values of the magnetic number for Pr=0.71, Da=0.1, G=0.1, Ec=0.5, Bi=0.2

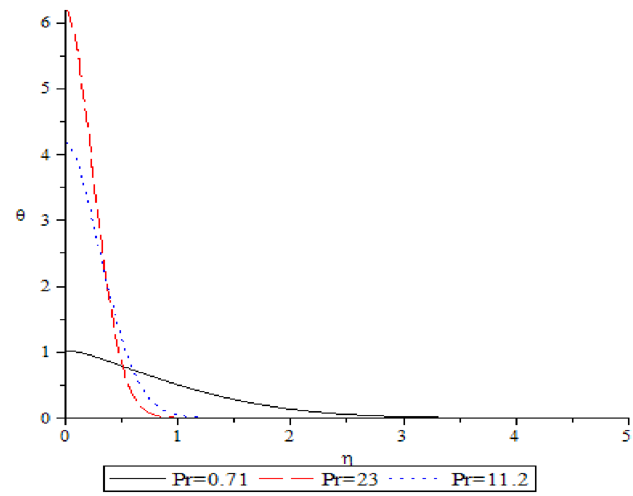

Figure 6: Temperature profile for various values of the Prandtl number for G=0.1, Da=0.1, H=1, Ec=0.5, Bi=0.2

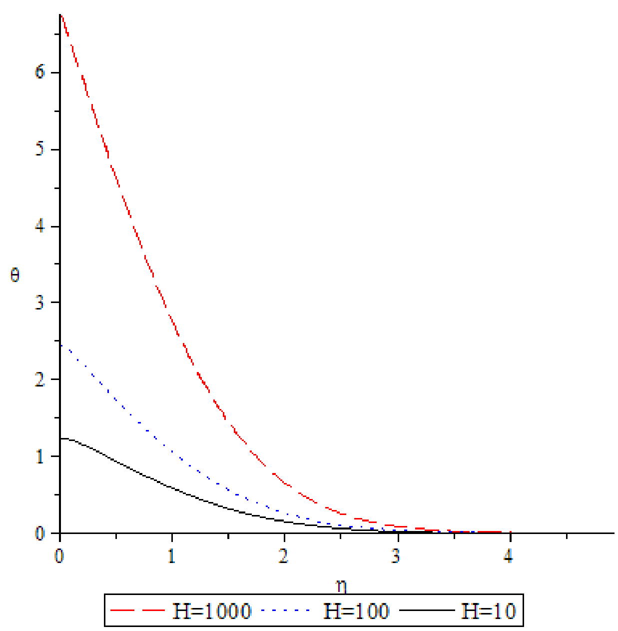

Figure7: Temperature profile for various values of the magnetic number for Pr=0.71, Da=0.1, H=1, Ec=0.5, Bi=0.2

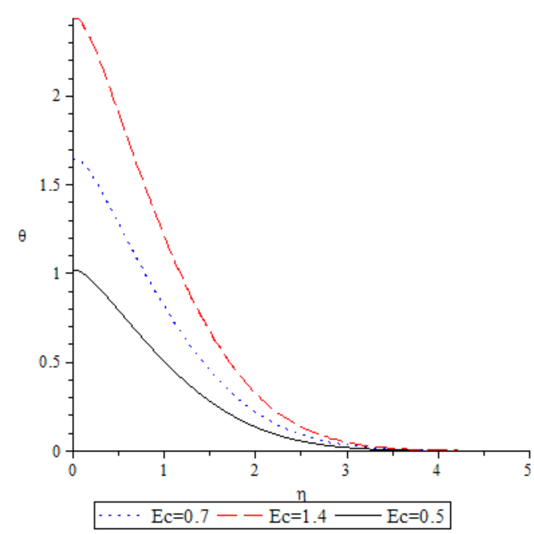

Figure 8: Temperature profile for various values of the Eckert number for Pr=0.71, Da=0.1, H=1, G=0.1, Bi=0.**2**

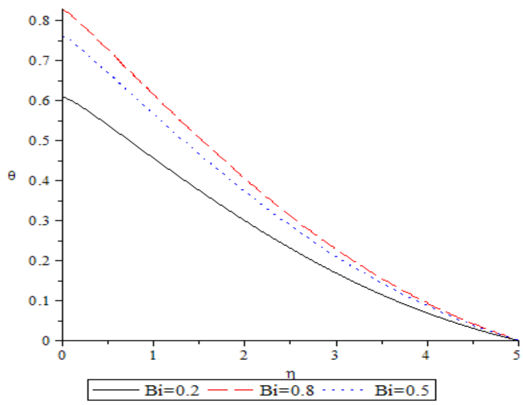

Figure 9: Temperature profile for various values of the Biot number for Pr=0.71, Da=0.1, H=1, Ec=0.5, G=0.1

Table 1. Computed values of the skin friction coefficient and Nusselt numbers.

G	Da	M	Pr	Bi	Ec	Skin friction	Nusselt Number
0.1	0.1	1	0.1	0.2	0.5	3.55241253040699	0.0779836823118050
0.1	0.1	1	0.1	0.5	0.5	3.55644638163061	0.1190323575662260
0.1	0.1	1	0.1	0.8	0.5	3.55821871450227	0.1370736088289950
0.3	0.1	1	0.1	0.2	0.5	3.58547028812497	0.0774659153210403
0.5	0.1	1	0.1	0.2	0.5	3.61880253439045	0.0769391243550531
0.1	0.3	1	0.1	0.2	0.5	2.43813764505099	0.0883674971916178
0.1	0.5	1	0.1	0.2	0.5	2.14702942368169	0.0909375149471808
0.1	0.1	2	0.1	0.2	0.5	3.69064463991404	0.0766560317414898
0.1	0.1	3	0.1	0.2	0.5	3.82387992298303	0.0753706342528905
0.1	0.1	1	0.71	0.2	0.5	3.56197739443892	-0.004435667293178
0.1	0.1	1	3	0.2	0.5	3.58245143033632	-0.205999032503159
0.1	0.1	1	0.1	0.2	0.8	3.55551525843022	0.0556210511332575
0.1	0.1	1	0.1	0.2	1.3	3.56070699186446	0.0181989358985879

CONCLUSION

The problem of viscous dissipation effect on the mixed convection MHD flow towards a stagnation point with convective boundary conditions has been studied in this research work. the coupled nonlinear Partial differential equations were converted to nonlinear coupled partial differential equations using similarity transformations and the solved numerically with the classical Runge-Kutta techniques together with shooting method.

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