

## ON THE NUMBER OF LIMIT CYCLES OF CERTAIN POLYNOMIAL SYSTEMS

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### ABSTRACT

*In this paper, we consider two dimensional autonomous system of the form:*

$$\left. \begin{aligned} \dot{x} &= P(x, y) \\ \dot{y} &= Q(x, y) \end{aligned} \right] \quad (A)$$

*in which  $P$  and  $Q$  are polynomials in  $x$  and  $y$ . We write the system A in the form of*

$$\begin{aligned} \dot{x} &= \lambda x + y + p_2(x, y) + p_3(x, y) \\ \dot{y} &= -x + \lambda y + q_2(x, y) + q_3(x, y) \end{aligned} \quad (B)$$

*Where  $p_2, q_2$  and  $p_3, q_3$  are homogeneous quadratic and cubic polynomials in  $x$  and  $y$ . The question of interest is the maximum possible number of limit cycles (a limit cycle is an isolated closed orbit) of such systems which can bifurcate out of the origin in terms of the degree of  $P$  and  $Q$ . It is second part of known Hilbert's sixteenth problem. Research on Hilbert's sixteenth problem in general usually proceeds but the investigation of particular classes of polynomial system. In this paper, in particular it is given that up to six limit cycles can bifurcate from fine focus of some examples of cubic system (B). Also we have given one example of quadratic system with at most one limit cycle.*

**Keywords:** *Limit cycles, perturbation, bifurcation, autonomous system, fine focus.*