MHD MICROPOLAR FLUID FLOW THROUGH VERTICAL PLATE WITH HEAT GENERATION

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ABSTRACT

The numerical studies are performed to examine the micropolar fluid flow past an infinite vertical heated generation in a magnetic field. Finite difference technique is used as a tool for the numerical approach. The micropolar fluid behavior on two- dimensional unsteady flow has been considered and its non similar solution have been obtained. No similar equations of the corresponding momentum, angular momentum, energy and continuity equations are derived by employing the usual transformations. The dimensionless non similar equations for momentum, angular momentum, energy equation and continuity equations are solved numerically by finite difference technique. The effects on the velocity, microrotation, the spin gradient viscosity, Prandtl number, Grashoff number and Eckert number of the various important parameters entering into the problem separately are discussed with the help of graphs.

Keywords: Magnetohydrodynamics(MHD), Micropolar Fluid, Heat Generation, Vertical Plate.

INTRODUCTION

Because of the increasing importance of materials flow in industrial processing and elsewhere and the fact shear behavior cannot be characterized by Newtonian relationships, a new stage in the evaluation of fluid dynamic theory is in the progress. Eringen(1966) proposed a theory of molecular fluids taking into account the internal characteristics of the subtractive particles, which are allowed to undergo rotation. Physically, the micropolar fluid can consists of a suspension of small, rigid cylindrical elements such as large dumbbell-shaped molecules. The theory of micropolar fluids is generating a very much increased interest and many classical flows are being re-examined to determine the effects of the fluid microstructure. The concept of micropolar fluid deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and micromotions of the fluids elements. These fluid contain dilute suspension of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia. Micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can effect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. It has many practical applications, for example analyzing the behavior of exotic lubricants, the flow of colloidal suspensions, polymetric fluids, liquid crystals, additive suspensions, human and animal blood, turbulent shear flow and so forth.

Peddision and McNitt(1970) derived boundary layer theory for micropolar fluid which is important in a number of technical process and applied this equations to the problems of steady stagnation point flow, steady flow past a semi-infinite flat plate. Eringen (1972) developed the theory of thermo micropolar fluids by extending the theory of micropolar fluids. The above mentioned work they have extended the work of El-Arabawy (2003) to a MHD flow taking into account the effect of free convection and micro rotation inertia term which has been neglected by El-Arabawy (2003). However, most of the previous works assume that the plate is at rest.

Quite recently, a numerical study of steady combined heat and mass transfer by mixed convection flow past a continuously moving infinite vertical porous plate under the action of strong magnetic field with constant suction velocity, constant heat and mass fluxes have been investigated by *Alam et. al*.(2008). For unsteady two dimensional case, the above problem becomes more complicated. These type of problems play a special role in nature, in many separation processes as isotope separation, in mixtures between gases, in many industrial applications as solidification of binary alloy as well as in astrophysical and geophysical engineering.

GOVERNING EQUATIONS:

The generalized Continuity equation, Momentum equation, Angular Momentum equation, Energy equation are together with the Ohm's law and Maxwell's equations form the basis of studying Magneto Fluid Dynamics (MFD) as follows:

The continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

The momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=F_x-\frac{1}{\rho}\frac{\partial P}{\partial x}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y}$$
$$+\frac{\sigma'}{\rho}\left\{\left(wB_xB_z-uB_z^{\,2}\right)-\left(uB_y^{\,2}-vB_xB_y\right)\right\} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=F_y-\frac{1}{\rho}\frac{\partial P}{\partial y}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial x}$$
$$+\frac{\sigma'}{\rho}\left\{\left(uB_xB_y-vB_x^{\,2}\right)-\left(vB_z^{\,2}-wB_yB_z\right)\right\} \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=F_z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$+\frac{\sigma'}{\rho}\left\{\left(vB_yB_z-wB_y^{\,2}\right)-\left(wB_x^{\,2}-uB_xB_z\right)\right\} \tag{4}$$

The angular momentum equation:

$$\frac{\partial \Gamma}{\partial t}+u\frac{\partial \Gamma}{\partial x}+v\frac{\partial \Gamma}{\partial y}+w\frac{\partial \Gamma}{\partial z}=\frac{\gamma}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\chi}{\rho j}\left(\frac{\partial^2 \Gamma}{\partial x^2}+\frac{\partial^2 \Gamma}{\partial y^2}+\frac{\partial^2 \Gamma}{\partial z^2}\right)-\frac{2\chi}{\rho j}\Gamma \tag{5}$$

The energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{6}$$

Considering an unsteady flow of a fluid along $y=0$ in a rotating system.

The continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{7}$$

The momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=g\beta(T-T_\infty)+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y}+ \frac{\sigma'}{\rho}\left\{\left(wB_xB_z-uB_z^{\,2}\right)-\left(uB_y^{\,2}-vB_xB_y\right)\right\} \tag{8}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial x}+ \frac{\sigma'}{\rho}\left\{\left(uB_xB_y-vB_x^{\,2}\right)-\left(vB_z^{\,2}-wB_yB_z\right)\right\} \tag{9}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+ \frac{\sigma'}{\rho}\left\{\left(vB_yB_z-wB_y^{\,2}\right)-\left(wB_x^{\,2}-uB_xB_z\right)\right\} \tag{10}$$

The angular momentum equation:

$$\frac{\partial\Gamma}{\partial t}+u\frac{\partial\Gamma}{\partial x}+v\frac{\partial\Gamma}{\partial y}+w\frac{\partial\Gamma}{\partial z}=\frac{\gamma}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\chi}{\rho j}\left(\frac{\partial^2\Gamma}{\partial x^2}+\frac{\partial^2\Gamma}{\partial y^2}+\frac{\partial^2\Gamma}{\partial z^2}\right)-\frac{2\chi}{\rho j}\Gamma \tag{11}$$

The energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{12}$$

Since the plate occupying the plane $y=0$ is of infinite extent and the fluid motion is steady all physical quantities will depends only upon x and y.

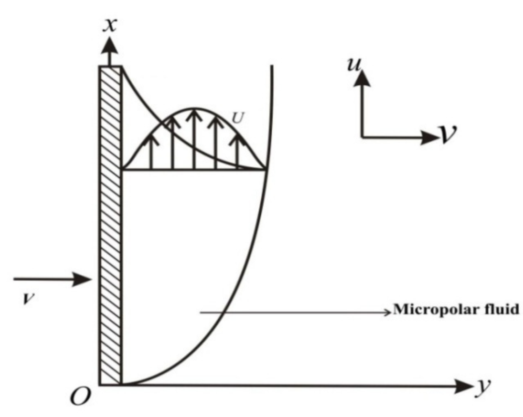

Figure 1. Boundary layer development on a vertical plate

Therefore the equations (7) to (12) becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{13}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\ +\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y}-\frac{\sigma' u B_0^2}{\rho} \tag{14}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\ +\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial x} \tag{15}$$

$$\frac{\partial\Gamma}{\partial t}+u\frac{\partial\Gamma}{\partial x}+v\frac{\partial\Gamma}{\partial y}=\frac{\gamma}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\chi}{\rho j}\left(\frac{\partial^2\Gamma}{\partial x^2}+\frac{\partial^2\Gamma}{\partial y^2}\right)-\frac{2\chi}{\rho j}\Gamma \tag{16}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{17}$$

Let, viscosity of the fluid be small and let δ be small thickness of the boundary layer. Let, $\varepsilon << 1$ be the order of magnitude of δ, i.e., $O(\delta)=\varepsilon$, Let the order of magnitude of u,χ and Γ are one i.e. $O(u)=1, O(\chi)=1, O(\Gamma)=1, O(T)=1$.
Then the order of magnitude of v and y are ε and the order of magnitude of t is one, i.e. $O(v)=\varepsilon, O(y)=\varepsilon \;\&\; O(t)=1$

Hence,

$$O\left(\frac{\partial u}{\partial x}\right)=1; O\left(\frac{\partial u}{\partial y}\right)=\frac{1}{\varepsilon};\ O\left(\frac{\partial^2 u}{\partial y^2}\right)=\frac{1}{\varepsilon^2}; O\left(\frac{\partial^2 u}{\partial x^2}\right)=1;\ O\left(\frac{\partial v}{\partial x}\right)=\varepsilon; O\left(\frac{\partial v}{\partial y}\right)=1;\ \ O\left(\frac{\partial^2 v}{\partial y^2}\right)=\frac{1}{\varepsilon};$$

$$O\left(\frac{\partial^2 u}{\partial x^2}\right)=\varepsilon^2;\ O\left(\frac{\partial\Gamma}{\partial x}\right)=1; O\left(\frac{\partial\Gamma}{\partial y}\right)=\frac{1}{\varepsilon};\ O\left(\frac{\partial^2\Gamma}{\partial x^2}\right)=1;\ \ O\left(\frac{\partial^2\Gamma}{\partial y^2}\right)=\frac{1}{\varepsilon^2};\ O\left(\frac{\partial T}{\partial x}\right)=1;$$

$$O\left(\frac{\partial T}{\partial y}\right)=\frac{1}{\varepsilon};\ O\left(\frac{\partial^2 T}{\partial x^2}\right)=1;\ \ O\left(\frac{\partial^2 T}{\partial y^2}\right)=\frac{1}{\varepsilon^2};\ O\left(g\beta(T-T_\infty)\right)=1$$

$$O\left(\upsilon+\frac{\chi}{\rho}\right)=\varepsilon^2; O\left(\frac{\chi}{\rho}\right)=\varepsilon;\ O\left(\frac{\gamma}{\rho j}\right)=\varepsilon^2; O\left(\frac{\chi}{\rho j}\right)=\varepsilon;\ O\left(\frac{\sigma' B_0^2}{\rho}\right)=1; O\left(\frac{k}{\rho c_p}\right)=\varepsilon^2.$$

Since the viscosity is very small, so neglecting the small order terms with the absence of the external force, we have from equations (13)-(17)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{18}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)\ +\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y}-\frac{\sigma' u B_0^2}{\rho} \tag{19}$$

$$\frac{\partial\Gamma}{\partial t}+u\frac{\partial\Gamma}{\partial x}+v\frac{\partial\Gamma}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2\Gamma}{\partial y^2}-\frac{\chi}{\rho j}\frac{\partial u}{\partial y} \tag{20}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{21}$$

The boundary conditions for the problem are:
at $\tau=0;\ u=0;\ v=0;\ \Gamma=0;\ T=0$ every where

$$\text{at}\ \tau=0 \left\{\begin{array}{l} u=0;\ v=0;\ \Gamma=0;\ T=0\ \ at\ \ x=0 \\ u=0;\ \ v=0;\ \ \Gamma=1;\ \ T=1\ \ at\ \ \ y=0 \\ u=0;\ \ v=0;\ \ \Gamma=0;\ \ T=0\ \ at\ \ \ y\to\infty \end{array}\right\} \tag{22}$$

MATHEMATICAL FORMULATION

Since the solutions of the governing equations (18)-(21) under the initial conditions (22) will be based on the finite difference method it is required to make the said equations dimensionless. For this purpose we now introduce the following dimensionless quantities

$$X = x\frac{U_0}{\upsilon},\ Y = y\frac{U_0}{\upsilon},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0}\ ,\ \tau = t\frac{U^2{}_0}{\upsilon},\ \Gamma = \overline{\Gamma}\frac{U^2{}_0}{\upsilon},\ T = \overline{T}\left(T_w - T_\infty\right)$$

For the above dimensionless variable we have

$u = UU_0,\ v = VU_0$ Using these relations we have the following derivatives

$$\frac{\partial u}{\partial t} = \frac{U_0^3}{\upsilon}\frac{\partial U}{\partial \tau}\ ,\quad \frac{\partial u}{\partial x} = \frac{U_0^2}{\upsilon}\frac{\partial u}{\partial X}\ ,\quad \frac{\partial u}{\partial y} = \frac{U_0^2}{\upsilon}\frac{\partial U}{\partial Y}\ ,\quad \frac{\partial v}{\partial y} = \frac{U_0^2}{\upsilon}\frac{\partial V}{\partial Y}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U_0^3}{\upsilon^2}\frac{\partial^2 U}{\partial Y^2}\ ,\quad \frac{\partial \Gamma}{\partial t} = \frac{U_0^3}{\upsilon^2}\frac{\partial \overline{\Gamma}}{\partial \tau}\ ,\quad \frac{\partial \Gamma}{\partial x} = \frac{U_0^2}{\upsilon^2}\frac{\partial \overline{\Gamma}}{\partial X}\ ,\quad \frac{\partial \Gamma}{\partial y} = \frac{U_0^2}{\upsilon^2}\frac{\partial \overline{\Gamma}}{\partial Y}$$

$$\frac{\partial^2 \Gamma}{\partial y^2} = \frac{U_0^3}{\upsilon^3}\frac{\partial \overline{\Gamma}}{\partial Y^2}\ ,\quad \frac{\partial T}{\partial t} = \frac{U_0^2\left(T_w - T_\infty\right)}{\upsilon}\frac{\partial \overline{T}}{\partial \tau}\ ,\quad \frac{\partial T}{\partial x} = \frac{U_0\left(T_w - T_\infty\right)}{\upsilon}\frac{\partial \overline{T}}{\partial X}$$

$$\frac{\partial T}{\partial y} = \frac{U_0\left(T_w - T_\infty\right)}{\upsilon}\frac{\partial \overline{T}}{\partial Y}\ ,\quad \frac{\partial^2 T}{\partial y^2} = \frac{U^2{}_0\left(T_w - T_\infty\right)}{\upsilon}\frac{\partial^2 \overline{T}}{\partial Y^2}$$

The continuity equation :

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{23}$$

The momentum equation:

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} = \left(1+\Delta\right)\left(\frac{\partial^2 U}{\partial Y^2}\right) + \Delta\frac{\partial \overline{\Gamma}}{\partial Y} + G_r\overline{T} - \frac{\sigma' U B_0^2}{\rho\upsilon} \tag{24}$$

The angular momentum equation:

$$\frac{\partial \overline{\Gamma}}{\partial \tau} + U\frac{\partial \overline{\Gamma}}{\partial X} + V\frac{\partial \overline{\Gamma}}{\partial Y} = \Lambda\left(\frac{\partial^2 \overline{\Gamma}}{\partial Y^2}\right) - \lambda\left(\frac{\partial U}{\partial Y}\right) \tag{25}$$

Energy equation:

$$\frac{\partial \overline{T}}{\partial \tau} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \frac{1}{P_r}\left(\frac{\partial^2 \overline{T}}{\partial Y^2}\right) \tag{26}$$

Boundary conditions

$$\text{at},\ \tau=0, U=0, V=O,\ \overline{\Gamma}=0,\ \overline{T}=0\ \text{everywhere} \tag{27}$$

$$\text{at,} \quad \tau > 0 \quad \begin{Bmatrix} U = 0, V = 0, \overline{\Gamma} = 0, \ \overline{T} = 0 \ at \ X = 0 \\ U = 0, V = 0, \overline{\Gamma} = 1, \ \overline{T} = 1 \ at \ Y = 0 \\ U = 0, V = 0, \overline{\Gamma} = 0, \ \overline{T} = 0 \ at \ Y \to \infty \end{Bmatrix} \tag{28}$$

NUMERICAL SOLUTIONS

Now we attempt to solve the governing second order nonlinear coupled dimensionless partial differential equations with the associated initial and boundary conditions. The only difference between the two methods is that the implicit finite difference method being unconditionally stable is less expansive from the point of view of computer time. From the concept of the above discussion, for simplicity the explicit finite difference method has been used to solve equations (23)-(26) subject to the conditions given by (27) and (28).To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axis is taken along the plate and Y-axis is normal to the plate.

Here we consider that the plate of height $X_{\max} = 100$ i.e. X varies from 0 to 100 and regard $Y_{\max} = 25$ as corresponding to $\mathrm{Y} \to \infty$ i.e. Y varies from 0 to 25. There are m=125 and n=125 grid spacing in the X and Y directions respectively as shown in figure below:

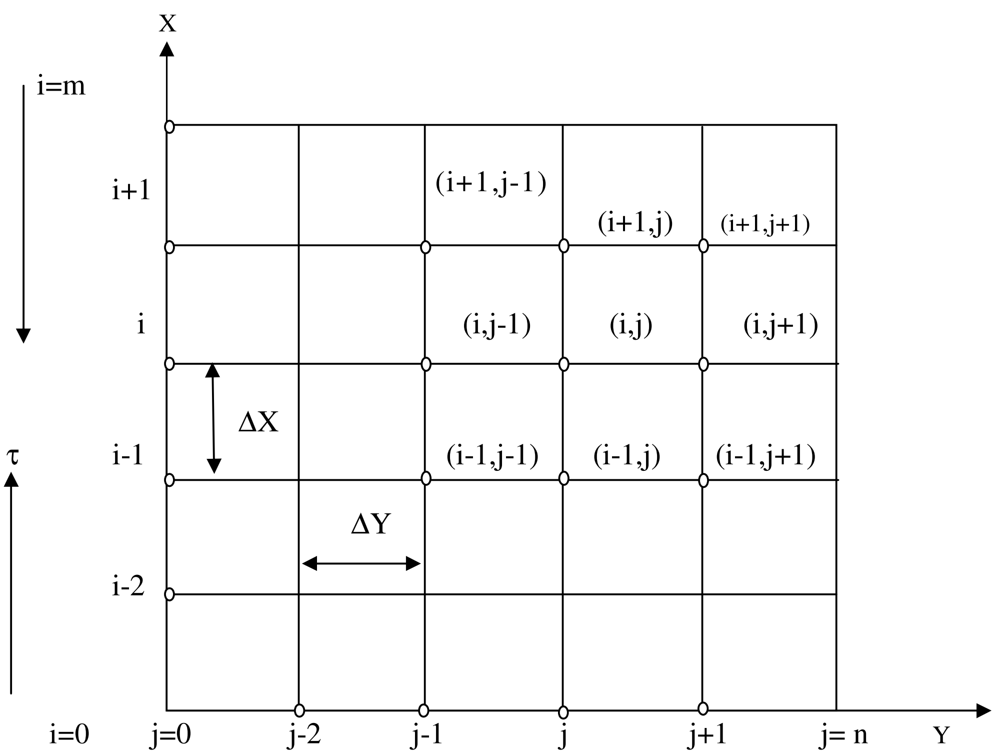

Figure 2. Finite difference space grid.

It is assumed that $\Delta \mathrm{X}, \Delta \mathrm{Y}$ are constant mesh sizes along X and Y directions respectively and taken as follows, $\Delta \mathrm{X}{=}0.8(0 \le x \le 100)$

$$\Delta Y{=}0.2(0 \le y \le 25)$$

with the smaller time-step, $\Delta\tau = 0.05$ Now, U', V' & Γ' denote the values of U,V & Γ at the end of time step respectively . Using the explicit finite difference approximation. We have,

Continuity equation:

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{29}$$

Momentum equation:

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = (1+\Delta)\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + \Delta\frac{\overline{\Gamma}_{i,j+1} - \overline{\Gamma}_{i,j}}{\Delta Y} + G_r\overline{T} - MU_{i,j}$$

(30)

Angular momentum equation:

$$\frac{\overline{\Gamma}'_{i,j} - \overline{\Gamma}_{i,j}}{\Delta\tau} + U_{i,j}\frac{\overline{\Gamma}_{i,j} - \overline{\Gamma}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{\Gamma}_{i,j+1} - \overline{\Gamma}_{i,j}}{\Delta Y} = \Lambda\frac{\overline{\Gamma}_{i,j+1} - 2\overline{\Gamma}_{i,j} + \overline{\Gamma}_{i,j-1}}{(\Delta Y)^2} \lambda\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \tag{31}$$

Energy equation:

$$\frac{\overline{T}'_{i,j} - \overline{T}_{i,j}}{\Delta\tau} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \frac{1}{P_r}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2}$$

(32)

And the initial and boundary conditions with the finite difference scheme are

$$U^0_{i,j} = 0,\ V^0_{i,j} = 0,\ \overline{\Gamma}^0_{i,j} = 0,\ \overline{T}^0_{i,j} = 0$$

$$U^n_{0,j} = 0,\ V^n_{0,j} = 0,\ \overline{\Gamma}^n_{0,j} = 0,\ \overline{T}^n_{0,j} = 0$$

$$U^n_{i,0} = 0,\ V^n_{i,0} = 0,\ \overline{\Gamma}^n_{i,0} = 1,\ \overline{T}^n_{i,0} = 1$$

$$U^n_{i,L} = 0,\ V^n_{i,L} = 0,\ \overline{\Gamma}^n_{i,L} = 0,\ \overline{T}^n_{i,L} = 0$$

Where, $L \to \infty$

Here the subscripts i and j designate the grid points with *x* and *y* coordinates respectively. During any one time-step, the coefficients $U_{i,j}$ and $V_{i,j}$ appearing in equations (29)-(32) are treated as constants. Then the end of anytime step $\Delta\tau$, the new velocity U' and V', at all interior nodal points may be obtained by successive applications of equations (29) and (30) respectively. This process is repeated in time and provided the time-step is sufficiently small, U and V should eventually converge to values which approximate the steady state solution of equations (23)-(27).

RESULTS

In this report, the effects of unsteady micropolar fluid behavior on a heated plate have been investigated using the finite difference technique. To study the physical situation of this problem, we have computed the numerical values by finite difference technique of velocity, micrirotation and temperature effect at the plate. It can be seen that the solutions are affected by the parameters namely, Microrotation parameter (Δ), Spin gradient viscosity parameter (Λ), Magnetic parameter (M), the vortex viscosity parameter (λ), Grashof Number (G_r) and Prandtl number (P_r). The main goal of the computation is to obtain the steady state solutions for the non-dimensional velocity $\boldsymbol{U}$, microrotation Γ' and temperature T' for different values of Microrotation parameter (Δ), Spin gradient viscosity parameter (Λ), the vortex viscosity parameter (λ), Grashof Number (G_r), Prandtl number (P_r).

DISCUSSION

For these computations the results have been calculated and presented graphically by dimensionless time $\tau = 10$ up to $\tau = 80$. The results of the computations show little changes for $\tau = 10$ to $\tau = 60$. But while arising at $\tau = 70$ and 80 the results remain approximately same but microrotation. Thus the solution for $\tau = 80$ are become steady-state. Moreover, the steady state solutions for transient values of U, Γ' and T' are shown in figures (3-26), for time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. The values of Microrotation parameters fixed. Whereas, figures (3-10) show the velocity profile for different values of Grashof Number $(G_r = 0.2, 0.4, 0.6)$ at time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. From this figures it is observed that the velocity profile increase with the increase of Grashof Number (G_r), and the velocity profiles are going upward direction. While arising at $\tau = 70$ and 80 the solutions become steady-state. Other important effects of microrotations are shown in figures (11-18) for different values of Spin gradient viscosity parameter (Λ) at time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. It is seen from this figures that the microrotaion increases with the increase of Spin gradient viscosity parameter (Λ) and is going to the upward direction from the horizontal wall with the increase of time. While arising at $\tau = 70$ and 80 the results also increasing with time. Other effects of temperature are shown in figures (19-26) for different values of Prandtl number $(P_r = 0.71, 1.0, 7.0)$ at time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. One the other hand, at salt water the Prandtl number is $(P_r = 1.0)$. It is seen from this figures that the temperature distribution is decreases with the increase of Prandtl number and the flow pattern is directed to the outer wall with the increase of time. While arising at $\tau = 70$ and 80 the flow becomes steady state.

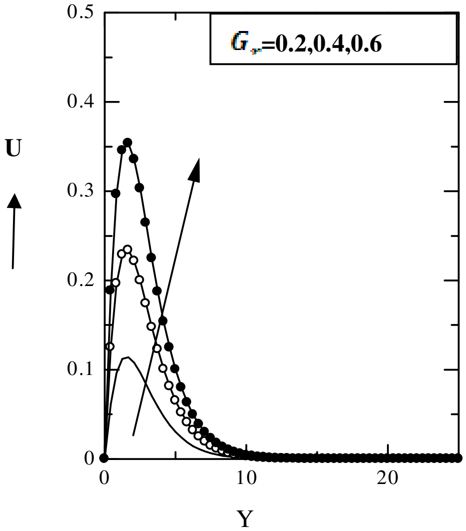

Figure 3. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 10

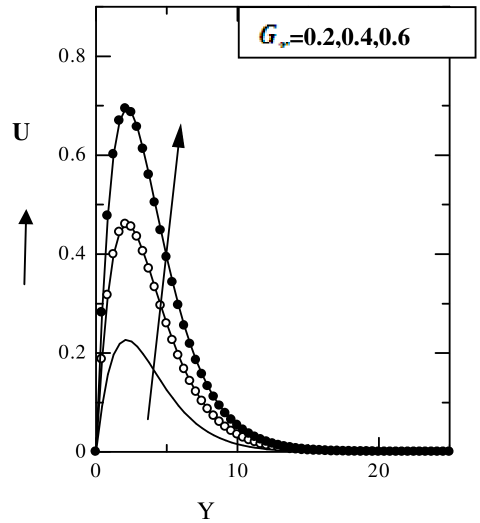

Figure 4. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 20

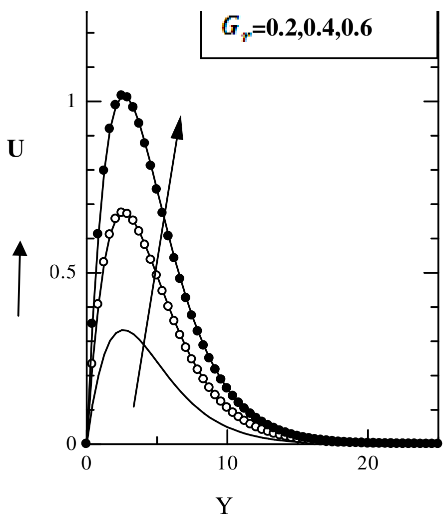

Figure 5. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 30

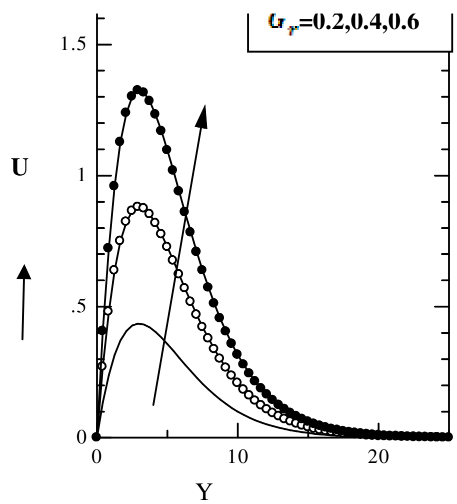

Figure 6. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 40

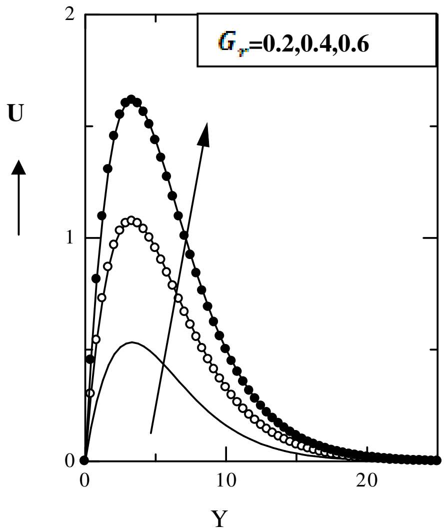

Figure 7. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01,M = 0.02 at time τ = 50

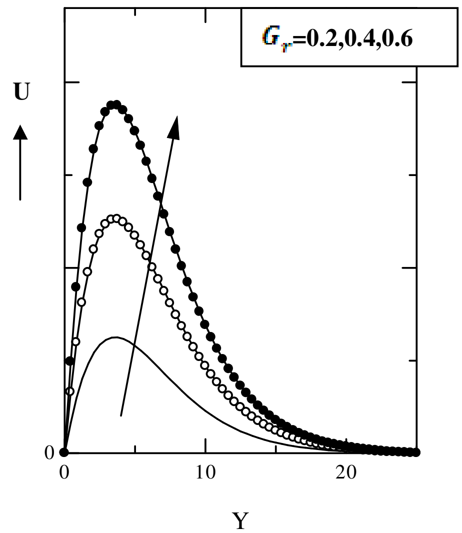

Figure 8. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 60

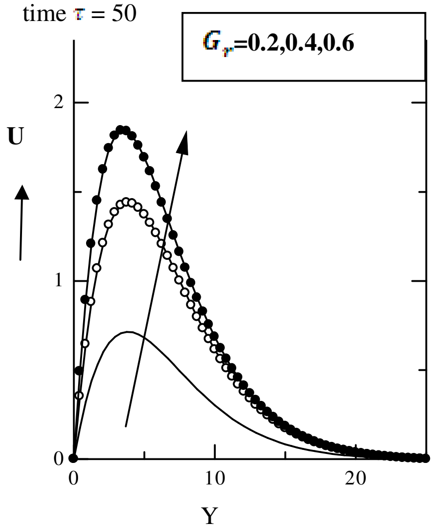

Figure 9. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 70

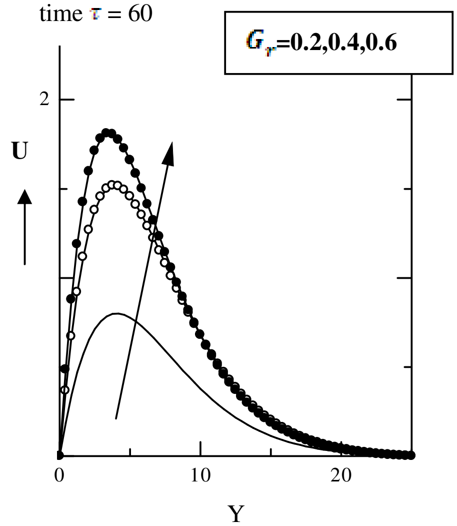

Figure 10. Velocity Profile for different values of Grashoff Number (G_r) and Δ = 0.01, M = 0.02 at time τ = 80

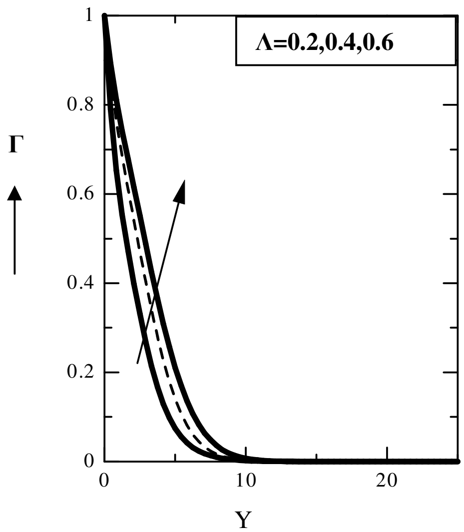

Figure 11. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 10

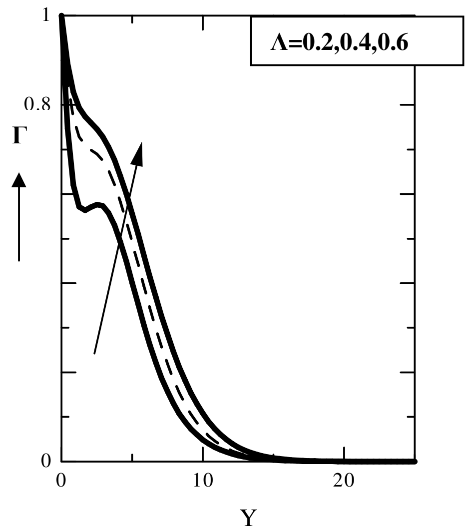

Figure12. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 20

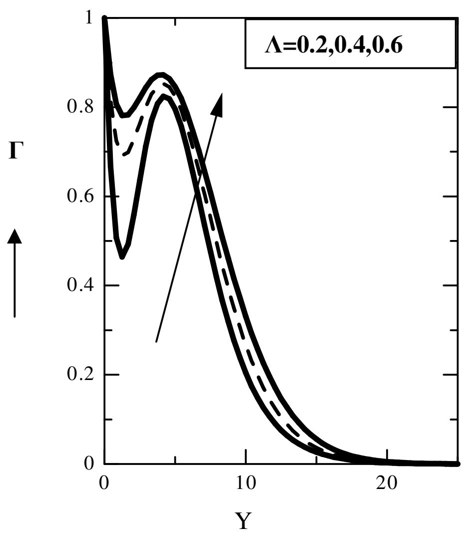

Figure 13. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 30

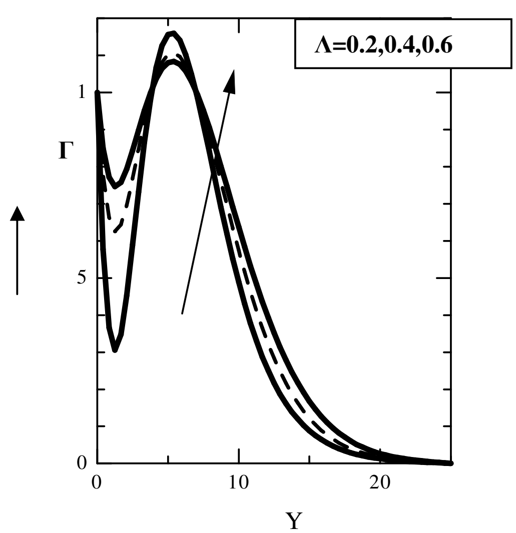

Figure 14. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 40

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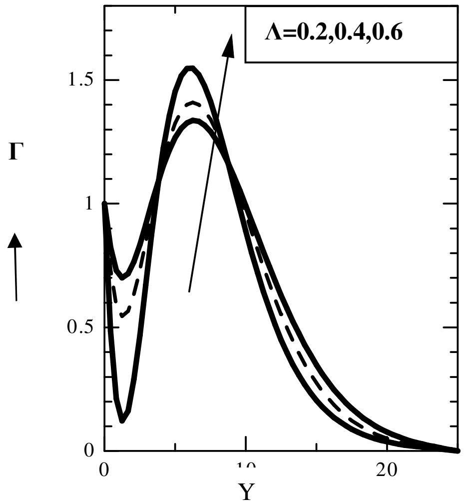

Figure 15. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 50

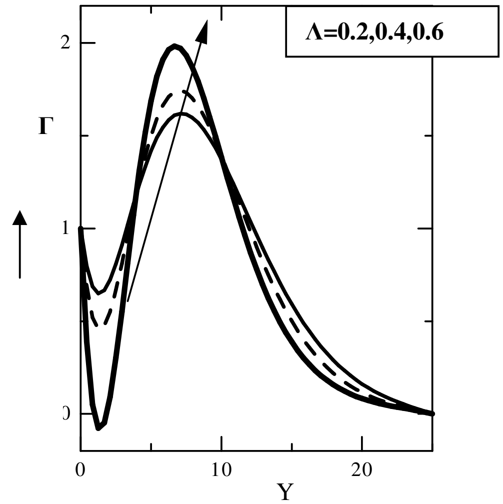

Figure 16. Microrotation Profile for different values of Spin Gradient viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 60

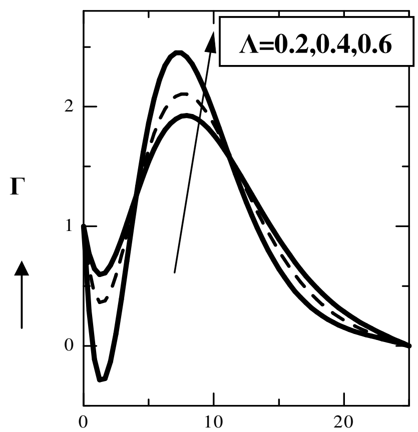

Figure17. Micr Y ion Profile for different values of Spin G it viscosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 70

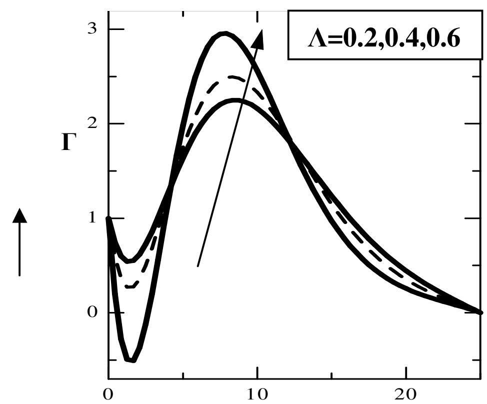

Figure 18. Microrota Y Profile for different values of Spin Gradie cosity parameter(Λ) and Δ = 0.01, M = 0.02 at time τ = 80

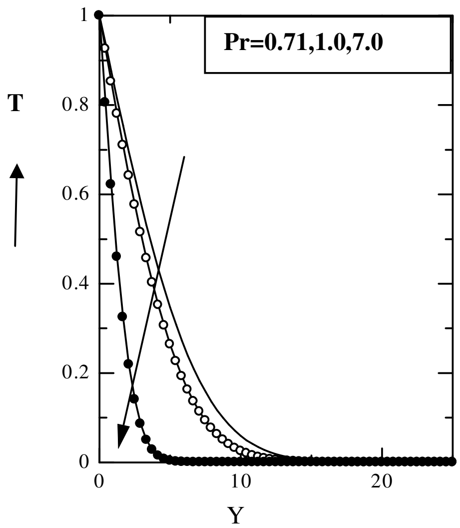

Figure 19. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 10

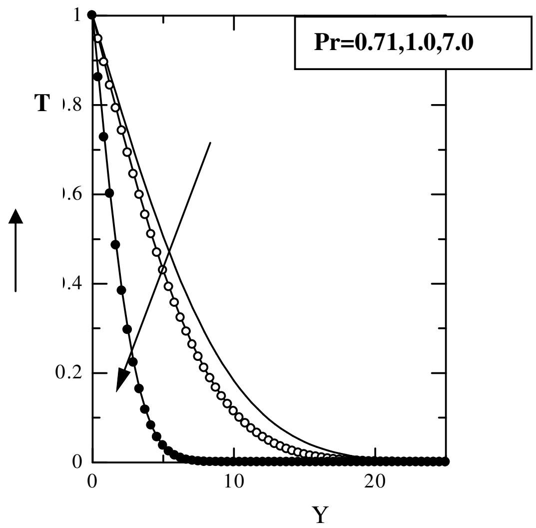

Figure 20. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 20

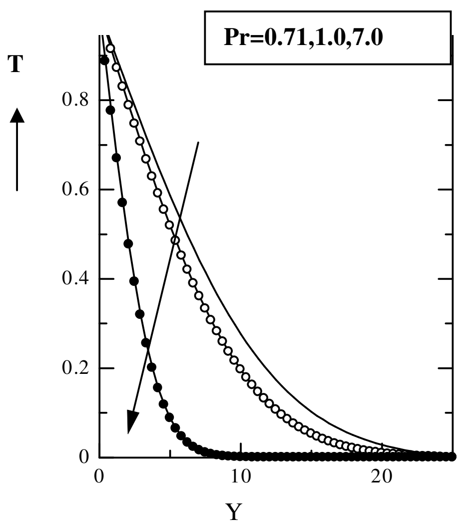

Figure 21. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 30

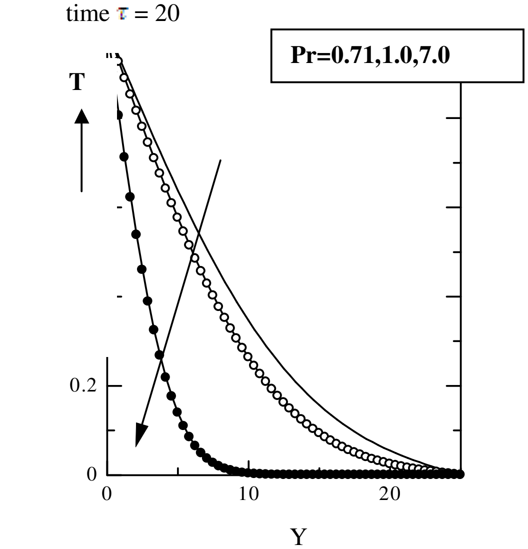

Figure 22. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 40

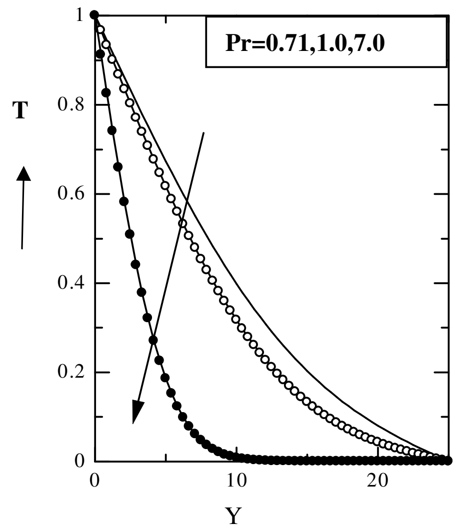

Figure 23. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 50

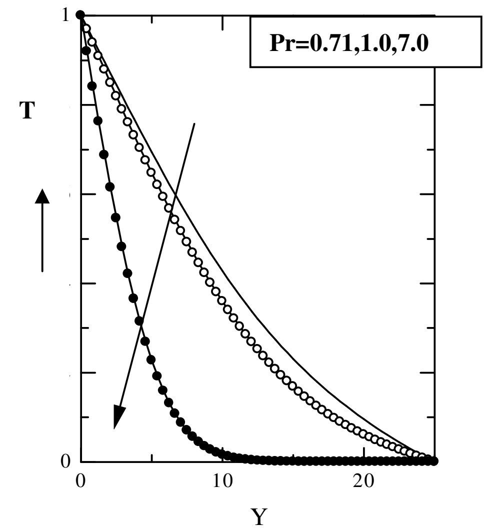

Figure 24. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 60

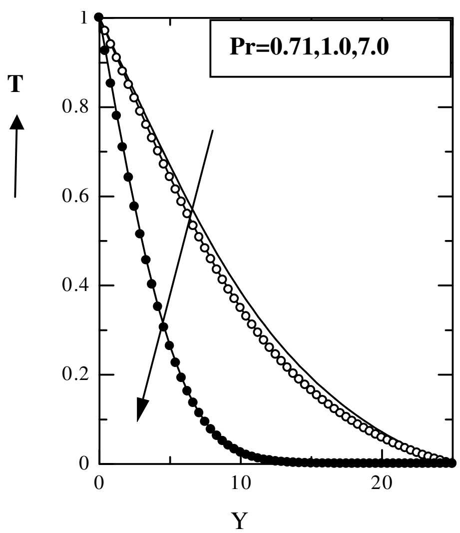

Figure 25. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 70

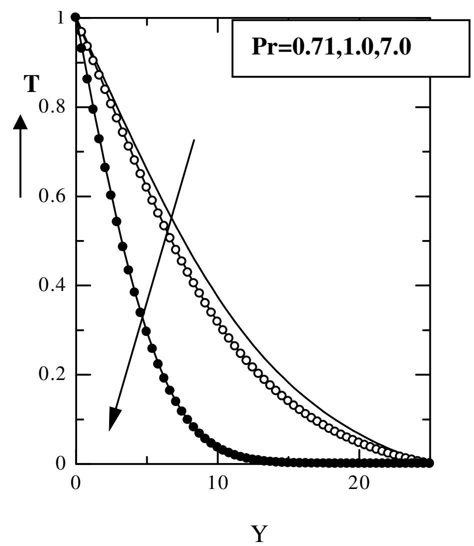

Figure 26. Temperature Profile for different values of Prandtl Number (P_r) and M = 0.02 at time τ = 80

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