CHARACTERISTIC CONSTITUTIVE LAWS OF CONCRETE UNDER MONOTONIC COMPRESSION LOADING

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ABSTRACT

This paper reviews the existing characteristic models for stress-strain relationship of concretes subjected to compression monotonic loading. Some of the models are based on the experimental investigation while some are based on analytical investigations. Some models are modified by other researchers to include some aspects that were neglected. Some of the models are proposed for the confined or unconfined concretes or even for both.

Keywords: Monotonic loading, *stress-strain relationship, confined concrete, unconfined concrete*

INTRODUCTION

Concrete is a composite material having a mixture of water, cement, fine and coarse aggregates sometimes with the addition of admixtures to obtain certain properties. The performance of concrete relies upon the nature of the constituent materials. The knowledge of stress-strain behavior and fracture mechanism of concrete constituents is needed to forecast how concrete behaves under various load types (Washa, 1950). Concrete is used in the various structural application and such structures are subjected to various compressive and tensile forces, this can be in a form of monotonic or sustained loading. Creep occurs as a result of sustained loading; a slow but continuous process leading to deformation of concrete (Youssef and Moftah, 2007). Studying the stress-strain diagram of reinforced concrete (RC) elements subjected to compression make it possible to determine the properties of RC members such as ductility and strength (Chen, 2013). This knowledge can only be obtained through testing and monitoring so that the performance can be studied and new theories can be introduced (Washa, 1950). However, in the research performed by Claisse and Dean (2013) and Martin and Terry (1991) shows that a sustained load producing low stresses applied prior to testing monotonically, and significantly increases the initial modulus of elasticity of concrete. This implies that creep, at least at low stresses, is not a result of detrimental cracking or damage to the material. It is proven that a maintained load creating low stresses connected prior to testing monotonically, and fundamentally expands the underlying modulus of elasticity of concrete. This infers creep at low stresses, is not an aftereffect of adverse splitting or harm to the material.

Recently, the introduction of many understandable and more complex models under numerous stress states to explain the behavior of concrete has been on the increase. However, most of these developed models have larger theoretical significance than practical importance but they solely explain some selected parts of concrete behavior and their application is limited to the basic pragmatic application. Vital highlights of the constitutive model of concrete ought to incorporate not just the precise description of the real behavior of concrete yet, in addition, the clearness of formulation and proficient application in a vigorous and stable nonlinear state determination algorithm (Aslani and Jowkarmeimandi, 2012). Sadeghi (1994, 1995, 2001) has produced a finite element model to eliminate the issue attributed to scale impact named "biaxial bending column simulation (BBCS)" where the column models were assessed based on the practical and simulated test. Along these lines, the presented nonlinear law for stress-strain relationships regarding confined and unconfined concretes subjected to monotonic and cyclic loads were changed and approved.

Existing Characteristic Models for Concrete under Monotonic Loading

Descriptions of the findings of the researchers are reviewed. There are some elements of concrete stress-strain model under monotonic loading.

Models of Sadeghi and Nouban

The developed stress-strain models for confined concrete under compression by Sadeghi (1995, 2002 and 2014) and Sadeghi and Nouban (2010a, 2017a) are suitable in the simulation of beams, columns, and beam-columns subjected to monotonic loading. These models are validated by some experimental tests carried by them and other researchers on RC columns under axial load and mono-axial and biaxial bending moment. The developed models can be used together with finite element analysis to simulate a vast array of RC elements subjected to different kinds of loading condition.

The main formulas for constitutive laws for unconfined and confined concretes under monotonic loading proposed by Sadeghi and Nouban are given Eqs. (1) and (2). These laws are valid for concretes under the monotonic loading having the characteristic strengths within 20 MPa < f_c' < 50 MPa:

$$\sigma = \frac{f_c'}{A\,(\frac{\varepsilon}{\varepsilon_0})^2+B\left(\frac{\varepsilon}{\varepsilon_0}\right)+C+D\,(\frac{\varepsilon}{\varepsilon_0})^{-1}} \quad \text{(For unconfined concrete)} \qquad (1)$$

$$\sigma = \frac{f_{cc}'}{A_L\,(\frac{\varepsilon}{\varepsilon_{c0}})^2+B_L\left(\frac{\varepsilon}{\varepsilon_{c0}}\right)+C_L+D_L\,(\frac{\varepsilon}{\varepsilon_{c0}})^{-1}} \quad \text{(For confined concrete)} \qquad (2)$$

The formulas for f_{cc}' and ε_{c0} [18, 19] are given below:

$$f_{cc}' = f_c'\,(1.000 + 2.5a.\omega_w) \quad \text{(For } \sigma/f_c' < 0.05 \text{ or } a.\omega_w < 0.1) \qquad (3)$$

$$f_{cc}' = f_c'\,(1.125 + 1.25a.\omega_w) \quad \text{(For } \sigma/f_c' \geq 0.05 \text{ or } a.\omega_w \geq 0.1) \qquad (4)$$

$$\varepsilon_{c0} = \varepsilon_0\,\left(\frac{f_{cc}'}{f_c'}\right)^2 \qquad (5)$$

With: $$\omega_w = k\,\left(\frac{A_t}{b_{max}\,.\,S_t}\right)\left(\frac{f_{yt}}{f_c'}\right) \qquad (6)$$

$$a = a_n\,.\,a_s \qquad (7)$$

$$a_n = 1 - \frac{8}{3\eta} \quad \text{(P.S.: } a_n = 1 \text{ for circular sections)} \qquad (8)$$

$$a_s = (1 - \frac{S_t}{2\,b_0})^2 \quad \text{(For rectangular and circular sections)} \qquad (9)$$

$$a_s = (1 - \frac{S_t}{2b_0}) \quad \text{(when the spiral type of transverse reinforcement is used)} \qquad (10)$$

Where:

σ: The stress,

ε: The strain,

f_c': The compression characteristic strength of unconfined concrete,

f_{cc}': The compression characteristic strength of confined concrete,

ISSN: 2223-9944, e ISSN: 2223-9553
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ε_0: The strain related to f_c',

ε_{c0}: The strain related to f_{cc}',

A_t: The cross-sectional area of transverse reinforcement,

f_{yt}: The transverse reinforcement's yield strength,

b_{max}: The section's larger dimension,

S_t: The transverse reinforcements' longitudinal spacing,

a: The confinement efficiency factor,

a_n: The stirrups' form factor,

a_s: The stirrups' spacing factor, and

b_0: The maximum spacing between the main reinforcements.

The k and η factors employed in Eqs. (6) and (8) for different types of transverse reinforcement are specified in papers published by Sadeghi and Nouban which are listed in the Reference Section.

To find unknowns A, B, C and D values for unconfined concrete as well as A_L, B_L, C_L and D_L, values for confined concrete, the stress and strain at points L, P and Y for confined concrete and similar points for unconfined concrete are used. At the peak of the curves, the slopes are equal to zero (see Figure 1).

These constitutive laws are illustrated schematically in Fig. 1.

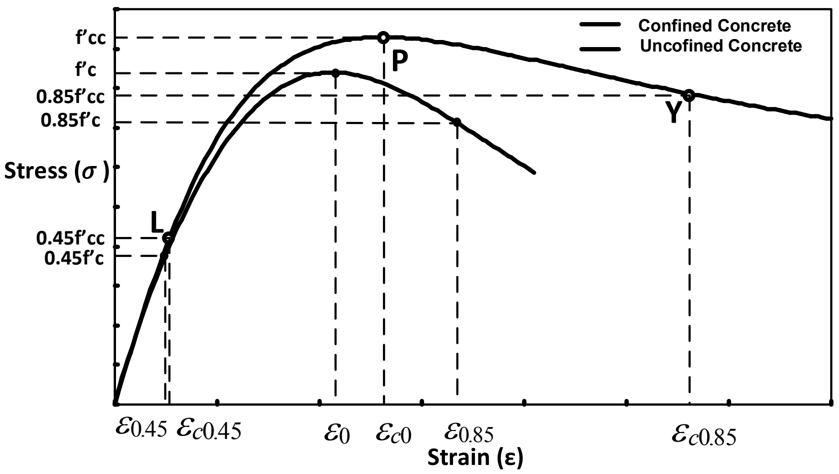

Figure 1. Stress-strain diagram of confined concrete subjected to monotonic loading (Kabir Sadeghi and Nouban, 2010)

For the definition of the symbols, please refer to the Notation Section at the end of this paper. The details of these formulas are given in the references: ((Sadeghi, 1995, 2002, 2014); (Sadeghi and Nouban, 2010a, 2017a) and the application of these formulas are given in the references: ((Sadeghi, 1998, 2011a, 2011b, 2015, 2017a, 2017b); (Sadeghi and Nouban, 2010b, 2017b, 2017c, 2018) and (Sadeghi and Sadeghi, 2013) and some similar References.

Models of Mander, Priestley and Park

Mander et al., (1989) proposed a technique that was used to show how both confined and unconfined concrete behaviors. They proposed a unified model for confined concrete subjected to monotonic compressive loading. This model is a streamlined variant of the

ISSN: 2223-9944, e ISSN: 2223-9553
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Karsan and Jirsa model shows that design the capacity of concrete to convey about tensile stresses. A stress-strain relationship is constructed using a single equation developed by the model. Due to its generalization, research and design have been presently adopted extensively by this approach.

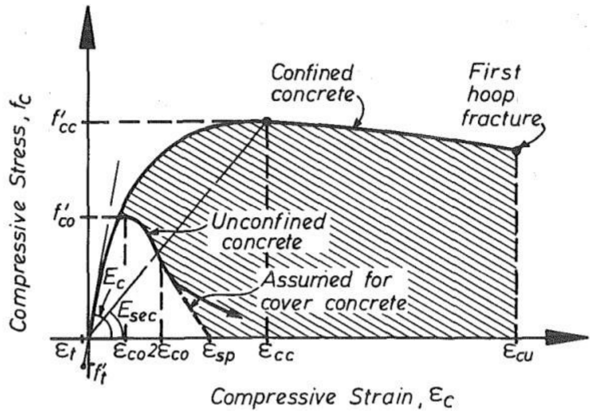

Figure 2. Stress-strain model for monotonic loading (Mander et al., 1989)

The details of the formulas given by Mander et al. are given in the references: (Mander et al., (1989); Karsan and Jirsa, (1969); Popovics (1973); Aslani and Jowkarmeimandi (2012)) and the application of these formulas are given in the references: (Youssef and Moftah (2007); Konstantinidis et al. (2004)).

Models of Hognestad

The model by Hognestad (1951) is generally being utilized for the stress-strain curve to symbolize the nature of normal strength and present it as an acceptable model for unconfined. In the model, stress-strain curve bend up to the pinnacle portion expected a second order parabola and part of the decrease is thought to be linear. The maximum stress, typically taken as 85 percent cylindrical strength of concrete i.e. $f_c = 0.85f_{ck}$ and the maximum compressive stress compared to strain (εc_0) is taken to be 0.002.

$$\sigma_c = f'_c\left[1 - 0.15\left(\frac{\varepsilon_c - \varepsilon'_c}{\varepsilon_u - \varepsilon'_c}\right)\right] \qquad \varepsilon'_c \leq \varepsilon_c \leq \varepsilon_u \tag{11}$$

$$\sigma_c = f'_c\left[\frac{2\varepsilon_c}{\varepsilon'_c} - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2\right] \qquad \varepsilon_c \leq \varepsilon'_c \tag{12}$$

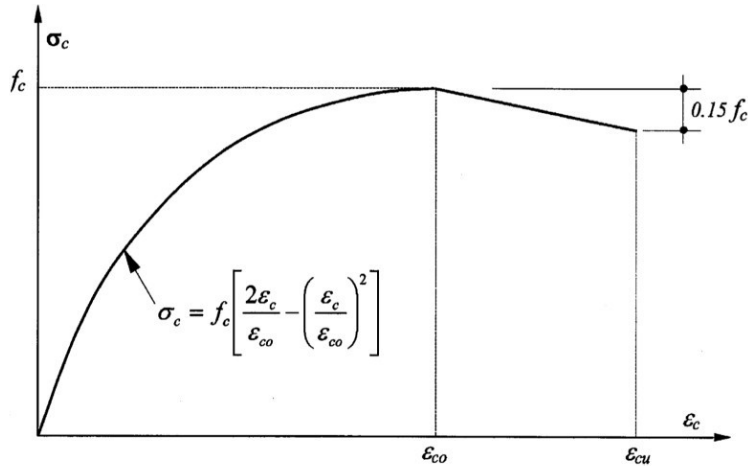

Figure 3. Stress-strain model under monotonic loading (Hognestad, 1951)

For the definition of the symbols, please refer to the Notation Section at the end of this paper. The details and applications of these formulas are given in the reference: Hognestad (1951).

Models of Kent and Park

In models proposed by Kent and Park, the climbing branch is symbolized by updating the Hognestad's 2nd-degree parabola changing $f_c = f_{ck}$, furthermore the part up to the highest point of the curve. They proposed the following equations for climbing and post-peak branches:

$$\sigma_c = f_c\left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002}\right)^2\right] \tag{13}$$

For the post-peak branch, a straight line having a slope that is characterized basically as function of strength concrete is proposed as given by the following equation:

$$\sigma_c = f_c\{1 - Z(\varepsilon_c - 0.002)\} \tag{14}$$

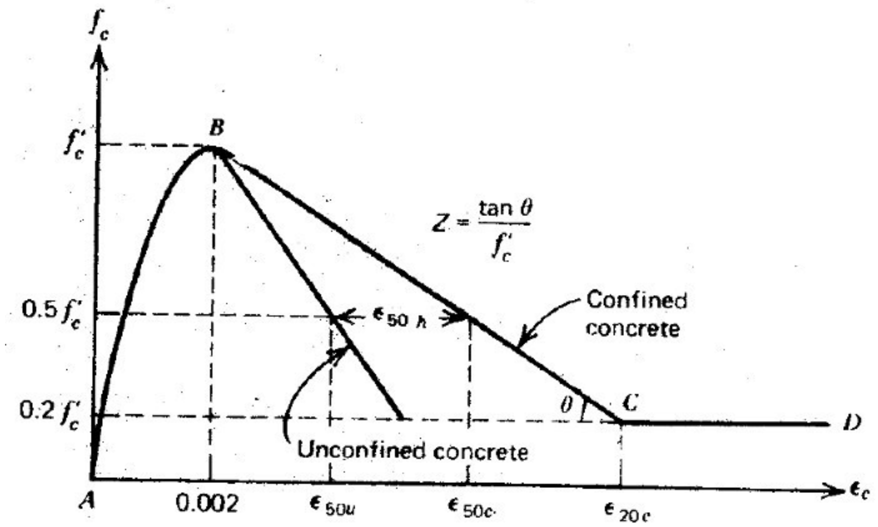

Figure 4. Stress-strain model under monotonic loading proposed by Kent et al. (1972)

For the definition of the symbols, please refer to the Notation Section at the end of this paper. The details of these formulas are given in the references: (Kent et al., (1972); Hognestad (1951)) and the application of these formulas are given in the reference: Carreira and Chu (1985).

CONCLUSION

In this paper, the famous characteristics constitutive laws applied on confined and unconfined concretes under monotonic loading are presented. Some of these models are not entirely original but modified from previous models taking into account some new or modified parameters. These models have been validated experimentally and analytically and are advised to be applied in calculation of stresses in confined and unconfined concretes in the RC structural members' sections in the numerical simulation of RC structures.

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