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MHD MICROPOLAR FLUID FLOW THROUGH VERTICAL POROUS MEDIUM

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ABSTRACT

The numerical studies are performed to examine the MHD micro-polar fluid flow through a vertical porous plate. Finite difference technique is used as a tool for the numerical approach. The micro-polar fluid behavior on two- dimensional unsteady flow has been considered and its non-similar solution have been obtained. Non-similar equations of the corresponding momentum, angular momentum and continuity equations are derived under MHD effect by employing the usual transformation. The dimensionless non-similar equations for momentum, angular momentum and continuity equations are solved numerically by finite difference technique. The effects on the velocity, micro rotation, the spin gradient viscosity, and vortex viscosity of the various important parameters entering into the problem separately are discussed with the help of graphs.

Keywords: MHD, Micropolar Fluid, Porous Medium.

INTRODUCTION

Because of the increasing importance of materials flow in industrial processing and elsewhere and the fact shear behavior cannot be characterized by Newtonian relationships, a new stage in the evaluation of fluid dynamic theory is in the progress. Eringen(1966) proposed a theory of molecular fluids taking into account the internal characteristics of the subtractive particles, which are allowed to undergo rotation. Physically, the micropolar fluid can consist of a suspension of small, rigid cylindrical elements such as large dumbbell-shaped molecules. The theory of micropolar fluids is generating a very much increased interest and many classical flows are being re-examined to determine the effects of the fluid microstructure. The concept of micropolar fluid deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and micromotions of the fluids elements. These fluid contain dilute suspension of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia. Micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can effect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. It has many practical applications, for example analyzing the behavior of exotic lubricants, the flow of colloidal suspensions, polymetric fluids, liquid crystals, additive suspensions, human and animal blood, turbulent shear flow and so forth.

Peddision and McNitt(1970) derived boundary layer theory for micropolar fluid which is important in a number of technical process and applied this equations to the problems of steady stagnation point flow, steady flow past a semi-infinite flat plate. Eringen (1972) developed the theory of thermo micropolar fluids by extending the theory of micropolar fluids. The above mentioned work they have extended the work of El-Arabawy (2003) to a MHD flow taking into account the effect of free convection and micro rotation inertia term which has been neglected by El-Arabawy (2003). However, most of the previous works assume that the plate is at rest.

Quite recently, a numerical study of steady combined heat and mass transfer by mixed convection flow past a continuously moving infinite vertical porous plate under the action of strong magnetic field with constant suction velocity, constant heat and mass fluxes have been investigated by *Alam et. al*.(2008). For unsteady two dimensional case, the above problem becomes more complicated. These type of problems play a special role in nature, in many separation processes as isotope separation, in mixtures between gases, in many industrial applications as solidification of binary alloy as well as in astrophysical and geophysical engineering.

THE BASIC GOVERNING EQUATION

Consider the unsteady flow of an electrically conducting fluid past an vertical porous plate coinciding with the plane y = 0 such as the x-axis is along the plate and the y-axis normal to it. An uniform magnetic field B_0 is applied in the direction of y-axis and the plate taken as a electrically non-conducting. Taking z-axis normal to xy-plane and assuming that the velocity **q** and the magnetic field **B** have the components (u, v, w) and (B_x, B_y, B_z) respectively.

The equation of continuity in vector form:

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho \mathbf{q}) = 0 \tag{1}$$

Equation of continuity in vector form for incompressible fluid:

$$\nabla\cdot\mathbf{q}=0 \tag{2}$$

In three-dimensional Cartesian coordinate system:

The equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3}$$

The equations of momentum

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=F_x-\frac{1}{\rho}\frac{\partial P}{\partial x}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y}$$
$$+\frac{\sigma'}{\rho}\left\{\left(wB_xB_z-uB_z^{\,2}\right)-\left(uB_y^{\,2}-vB_xB_y\right)\right\}-\frac{\upsilon}{K'}u \tag{4}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=F_y-\frac{1}{\rho}\frac{\partial P}{\partial y}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial x}$$
$$+\frac{\sigma'}{\rho}\left\{\left(uB_xB_y-vB_x^{\,2}\right)-\left(vB_z^{\,2}-wB_yB_z\right)\right\}-\frac{\upsilon}{K'}v \tag{5}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=F_z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$+\frac{\sigma'}{\rho}\left\{\left(vB_yB_z-wB_y^{\,2}\right)-\left(wB_x^{\,2}-uB_xB_z\right)\right\}-\frac{\upsilon}{K'}w \tag{6}$$

The equation of angular momentum is

$$\frac{\partial \Gamma}{\partial t}+u\frac{\partial \Gamma}{\partial x}+v\frac{\partial \Gamma}{\partial y}+w\frac{\partial \Gamma}{\partial z}=\frac{\chi}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\gamma}{\rho j}\left(\frac{\partial^2 \Gamma}{\partial x^2}+\frac{\partial^2 \Gamma}{\partial y^2}+\frac{\partial^2 \Gamma}{\partial z^2}\right)-2\frac{\chi}{\rho j}\Gamma \tag{7}$$

Thus in three-dimensional Cartesian coordinate system in the absence of external forces the continuity equation, the momentum equations and the angular momentum equation becomes

The Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{8}$$

The Momentum equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y}$$
$$+\frac{\sigma'}{\rho}\left\{\left(wB_xB_z-uB_z^{\,2}\right)-\left(uB_y^{\,2}-vB_xB_y\right)\right\}-\frac{\upsilon}{K'}u \tag{9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial x}$$
$$+\frac{\sigma'}{\rho}\left\{\left(uB_xB_y-vB_x^{\,2}\right)-\left(vB_z^{\,2}-wB_yB_z\right)\right\}-\frac{\upsilon}{K'}v \tag{10}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$+\frac{\sigma'}{\rho}\left\{\left(vB_yB_z-wB_y^{\,2}\right)-\left(wB_x^{\,2}-uB_xB_z\right)\right\}-\frac{\upsilon}{K'}w \tag{11}$$

The Angular momentum equation

$$\frac{\partial\Gamma}{\partial t}+u\frac{\partial\Gamma}{\partial x}+v\frac{\partial\Gamma}{\partial y}+w\frac{\partial\Gamma}{\partial z}=\frac{\chi}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\gamma}{\rho j}\left(\frac{\partial^2\Gamma}{\partial x^2}+\frac{\partial^2\Gamma}{\partial y^2}+\frac{\partial^2\Gamma}{\partial z^2}\right)-2\frac{\chi}{\rho j}\Gamma \tag{12}$$

Since the plate occupying the plane $y=0$ is of infinite extent and the fluid motion is steady, all physical equations will depend only upon *x* and *y*. Consider $B=(0,B_0,0)$ where B_0 is the uniform magnetic field acting normal to the plate. The boundary conditions for the problem are as follows:

$t=0,\ u=0,\ v=0,\ \Gamma=0$ at everywhere

$$t>0,\begin{cases}u=0,\ v=0,\ \Gamma=0\ \text{ at }\ x=0\\ u=0,\ v=0,\ \Gamma=1\ \text{ at }\ y=0\\ u=0,\ v=0,\ \Gamma=0\ \text{ at }\ y\to\infty\end{cases}$$

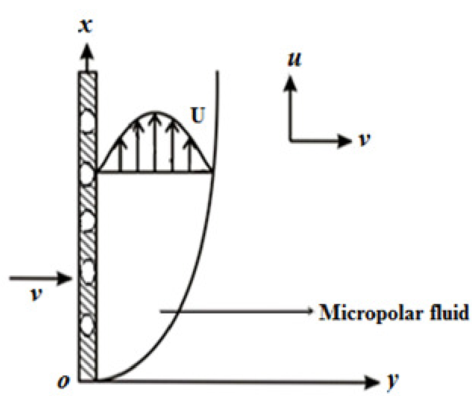

Figure 1. Boundary layer development on a vertical plate

Thus mathematically the problem reduces to a two dimensional problem. Then the equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{13}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y}-\frac{\upsilon}{K'}u-\frac{\sigma' u B_0^{\,2}}{\rho} \tag{14}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\left(\upsilon+\frac{\chi}{\rho}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial x}-\frac{\upsilon}{K'}v \tag{15}$$

$$\frac{\partial \Gamma}{\partial t}+u\frac{\partial \Gamma}{\partial x}+v\frac{\partial \Gamma}{\partial y}=\frac{\chi}{\rho j}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\gamma}{\rho j}\left(\frac{\partial^2 \Gamma}{\partial x^2}+\frac{\partial^2 \Gamma}{\partial y^2}\right)\ -2\frac{\chi}{\rho j}\Gamma \tag{16}$$

The viscosity of the fluid and the small thickness of the boundary layer δ are considered to be small. Let $\varepsilon << 1$ be the order of magnitude of δ, i.e. $O(\delta)=\varepsilon<<1$. Let the order of magnitude of u, x and Γ are one, i.e. $O(u)=1$, $O(x)=1$, $O(\Gamma)=1$. The order of magnitude of v and y are considered to be ε i.e., $O(v)=\varepsilon$, $O(y)=\varepsilon$ and $O(t)=1$.

Hence $O\left(\frac{\partial u}{\partial t}\right)=1,\quad O\left(\frac{\partial u}{\partial x}\right)=1,\quad O\left(\frac{\partial u}{\partial y}\right)=\frac{1}{\varepsilon},\quad O\left(\frac{\partial^2 u}{\partial x^2}\right)=1,\quad O\left(\frac{\partial^2 u}{\partial y^2}\right)=\frac{1}{\varepsilon^2}$

$$O\left(\frac{\partial v}{\partial t}\right)=\varepsilon,\quad O\left(\frac{\partial v}{\partial x}\right)=\varepsilon,\quad O\left(\frac{\partial v}{\partial y}\right)=1,\quad O\left(\frac{\partial^2 v}{\partial x^2}\right)=\varepsilon^2,\quad O\left(\frac{\partial^2 v}{\partial y^2}\right)=1$$

$$O\left(\frac{\partial \Gamma}{\partial t}\right)=1,\quad O\left(\frac{\partial \Gamma}{\partial x}\right)=1,\quad O\left(\frac{\partial \Gamma}{\partial y}\right)=\frac{1}{\varepsilon},\quad O\left(\frac{\partial^2 \Gamma}{\partial x^2}\right)=1,\quad O\left(\frac{\partial^2 \Gamma}{\partial y^2}\right)=\frac{1}{\varepsilon^2}$$

$$O\left(\upsilon+\frac{\chi}{\rho}\right)=\varepsilon^2,\ O\left(\frac{\chi}{\rho}\right)=\varepsilon,\ O\left(\frac{\upsilon}{K'}\right)=1,\ O\left(\frac{\gamma}{\rho j}\right)=\varepsilon^2,\ O\left(\frac{\chi}{\rho j}\right)=\varepsilon$$

within the boundary layer, so neglecting the small order terms, we have the following equations,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{17}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{\chi}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y}-\frac{\upsilon}{K'}u-\frac{\sigma' u B_0^{\,2}}{\rho} \tag{18}$$

$$\frac{\partial \Gamma}{\partial t}+u\frac{\partial \Gamma}{\partial x}+v\frac{\partial \Gamma}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 \Gamma}{\partial y^2}-\frac{\chi}{\rho j}\frac{\partial u}{\partial y} \tag{19}$$

With the boundary conditions

$t=0,\ u=0,\ v=0,\ \Gamma=0$ at everywhere

$$t>0,\begin{cases}u=0,\ v=0,\ \Gamma=0\ \text{ at }\ x=0\\ u=0,\ v=0,\ \Gamma=1\ \text{ at }\ y=0\\ u=0,\ v=0,\ \Gamma=0\ \text{ at }\ y\to\infty\end{cases}$$

where u, v are the velocity components in the x, y direction respectively, υ is the kinematics viscosity, ρ is the density. Also, K' is the permeability of the porous medium, Γ is the micro rotation component, χ is the vortex viscosity, γ is the spin gradient viscosity, j is the micro inertia per unit mass and other symbols have their usual meaning.

Since the solutions of the above governing equations under the initial conditions will be based on the finite difference method it is required to make the said equations dimensionless. For this purpose we now introduce the following dimensionless quantities:

$$X = \frac{xU_o}{\upsilon}, \quad Y = \frac{yU_o}{\upsilon}, \quad U = \frac{u}{U_o}, \quad V = \frac{v}{U_o}, \quad \tau = \frac{tU_o^2}{\upsilon}, \quad \Gamma = \Gamma' \frac{U_0^2}{\upsilon} \tag{20}$$

Using these relations we have the following derivatives

$$\frac{\partial u}{\partial t} = \frac{U_o^3}{\upsilon}\frac{\partial U}{\partial \tau}, \quad \frac{\partial u}{\partial x} = \frac{U_o^2}{\upsilon}\frac{\partial U}{\partial X}, \quad \frac{\partial u}{\partial y} = \frac{U_o^2}{\upsilon}\frac{\partial U}{\partial Y}, \quad \frac{\partial^2 u}{\partial y^2} = \frac{U_o^3}{\upsilon^2}\frac{\partial^2 U}{\partial Y^2}, \quad \frac{\partial v}{\partial y} = \frac{U_o^2}{\upsilon}\frac{\partial V}{\partial Y}, \quad \frac{\partial \Gamma}{\partial t} = \frac{U_o^3}{\upsilon}\frac{\partial \Gamma'}{\partial \tau},$$

$$\frac{\partial \Gamma}{\partial x} = \frac{U_o^2}{\upsilon}\frac{\partial \Gamma'}{\partial X}, \quad \frac{\partial \Gamma}{\partial y} = \frac{U_o^2}{\upsilon}\frac{\partial \Gamma'}{\partial Y}, \quad \frac{\partial^2 \Gamma}{\partial y^2} = \frac{U_o^3}{\upsilon^2}\frac{\partial^2 \Gamma'}{\partial Y^2}$$

In terms of the above dimensionless variables the nonlinear coupled partial differential equations are as follows

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{21}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = (1+\Delta)\frac{\partial^2 U}{\partial X^2} - \Delta\frac{\partial \Gamma'}{\partial X} - KU - MU \tag{22}$$

$$\frac{\partial \Gamma'}{\partial \tau} + U\frac{\partial \Gamma'}{\partial X} + V\frac{\partial \Gamma'}{\partial Y} = \Lambda\left(\frac{\partial^2 \Gamma'}{\partial X^2}\right) - \lambda\frac{\partial U}{\partial Y} \tag{23}$$

where $K = \dfrac{\upsilon^2}{k' U_0^2}$ (Permeability of porous medium);

$\Delta = \dfrac{\chi}{\rho\upsilon}$ (Micro rotation parameter);

$M = \dfrac{\sigma^2 B_0^2}{\rho}\dfrac{\upsilon}{U_0^2}$ (Magnetic parameter) ;

$\lambda = \dfrac{\chi\upsilon}{\rho j U_o^2}$ (Vortex viscosity parameter) and

$\Lambda = \dfrac{\gamma}{\upsilon\rho j}$ (Spin gradient viscosity parameter).

With the boundary conditions

$\tau = 0, \; U = 0, \; V = 0, \; \Gamma' = 0,$ every where

$$\tau > 0, \begin{cases} U = 0, \; V = 0, \; \Gamma' = 0 \text{ at } X = 0 \\ U = 0, \; V = 0, \; \Gamma' = 1 \text{ at } Y = 0 \\ U = 0, \; V = 0, \; \Gamma' = 0 \text{ at } Y \to \infty \end{cases}$$

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NUMERICAL SOLUTIONS

In this section, we attempt to solve the governing second order nonlinear coupled dimensionless partial differential equations with the associated initial and boundary conditions. For solving a transient free convection flow with mass transfer past a semi-infinite plate, Callahan and Marner(1976) used the explicit finite difference method which is conditionally stable. On the contrary, the same problem was studied by Soundalgekar and Ganesan(1980) by an implicit finite difference method which is unconditionally stable. The only difference method between the two methods is that the implicit method being unconditionally stable is less expansive from the point of view of computer time. However, these two methods respectively employed by Callahan and Marner(1976) and Soundalgekar and Ganesan(1980) produced the same results.

From the concept of the above discussion, for simplicity the explicit finite difference method has been used to solve equations (21)-(23) subject to its boundary conditions.

To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axis is taken along the plate and X-axis is normal to the plate. Here we consider that the plate of height $Y_{\max}(=100)$ i.e. X varies from 0 to 100 and regard $X_{\max}(=25)$ as corresponding to $Y\to\infty$ i.e. Y varies from 0 to 25. There are m = 125 and n = 125 grid spacing in the X and Y directions respectively as shown in figure 2.

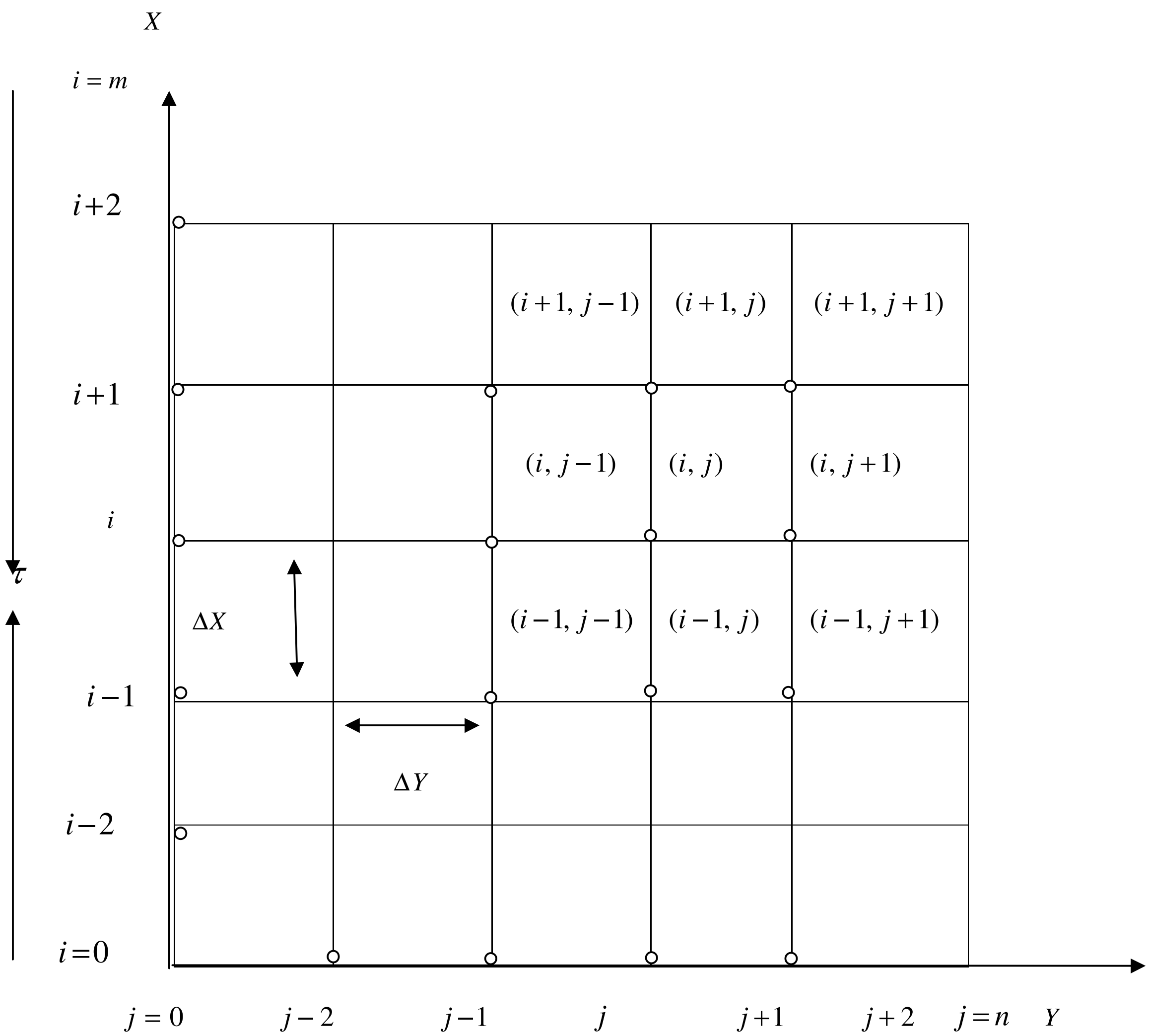

Figure 2. Finite difference space grid.

It is assumed that $\Delta X, \Delta Y$ are constant mesh sizes along X and Y directions respectively and taken $\Delta Y = 0.8(0 \le x \le 100)$, $\Delta X = 0.2(0 \le y \le 25)$ with the smaller time-step, $\Delta\tau = 0.005$.

Now $\bar{U}$ and $\bar{\Gamma}$ denote the values of U and Γ' at the end of a time-step respectively. Using the explicit finite difference approximation. we have

$$\left(\frac{\partial U}{\partial X}\right)_{i,j} = \frac{U_{i,j} - U_{i-1,j}}{\Delta X}; \left(\frac{\partial V}{\partial y}\right)_{i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y}, \left(\frac{\partial U}{\partial Y}\right)_{i,j} = \frac{U_{i,j+1} - U_{i,j}}{\Delta Y};$$

$$\left(\frac{\partial^2 U}{\partial Y^2}\right)_{i,j} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2}; \left(\frac{\partial \Gamma'}{\partial \tau}\right)_{i,j} = \frac{\bar{\Gamma}_{i,j} - \Gamma'_{i,j}}{\Delta\tau};$$

$$\left(\frac{\partial \Gamma'}{\partial x}\right)_{i,j} = \frac{\Gamma'_{i,j} - \Gamma'_{i-1,j}}{\Delta X}; \left(\frac{\partial \Gamma'}{\partial y}\right)_{i,j} = \frac{\Gamma'_{i,j+1} - \Gamma'_{i,j}}{\Delta Y}; \left(\frac{\partial^2 \Gamma'}{\partial Y^2}\right)_{i,j} = \frac{\Gamma'_{i,j+1} - 2\Gamma'_{i,j} + \Gamma'_{i,j-1}}{(\Delta Y)^2}$$

From the system of PDE with substituting the above relations into the corresponding differential equation we obtain an appropriate set of finite difference equation.

Continuity equation

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{24}$$

Momentum equation

$$\frac{\bar{U}_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j} - U_{i,j-1}}{\Delta Y} = (1+\Delta)\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + \Delta\frac{\Gamma'_{i,j+1} - \Gamma'_{i,j}}{\Delta Y} - KU_{i,j} - MU_{i,j} \tag{25}$$

Angular momentum equation

$$\frac{\bar{\Gamma}_{i,j} - \Gamma'_{i,j}}{\Delta\tau} + U_{i,j}\frac{\Gamma'_{i,j} - \Gamma'_{i-1,j}}{\Delta X} + V_{i,j}\frac{\Gamma'_{i,j+1} - \Gamma'_{i,j}}{\Delta Y} = \Lambda\frac{\Gamma'_{i,j+1} - 2\Gamma'_{i,j} + \Gamma'_{i,j-1}}{(\Delta Y)^2} - \lambda\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \tag{26}$$

the initial and boundary conditions with the finite difference scheme are

$U^0_{i,j} = 0, V^0_{i,j} = 0, \Gamma'^0_{i,j} = 0$

$U^n_{0,j} = 0, V^n_{0,j} = 0, \Gamma'^n_{0,j} = 0$

$U^n_{i,0} = 0, V^n_{i,0} = 0, \Gamma'^n_{i,0} = 1$

$U^n_{i,L} = 0, V^n_{i,L} = 0, \Gamma'^n_{i,L} = 0$

where $L \to \infty$.

Here the subscripts i and j designate the grid points with x and y coordinates respectively. During any one time-step, the coefficients $U_{i,j}$ and $V_{i,j}$ appearing in equations (24)-(26) are treated as constants. Then the end of anytime step $\Delta\tau$, the new velocity $\bar{U}$ and $\bar{V}$,at all interior nodal points nay be obtained by successive applications of equations (24) and (25) respectively. This process is repeated in time and provided the time-step is sufficiently small, U and V should eventually converge to values which approximate the steady state solution of equations (21)-(23).

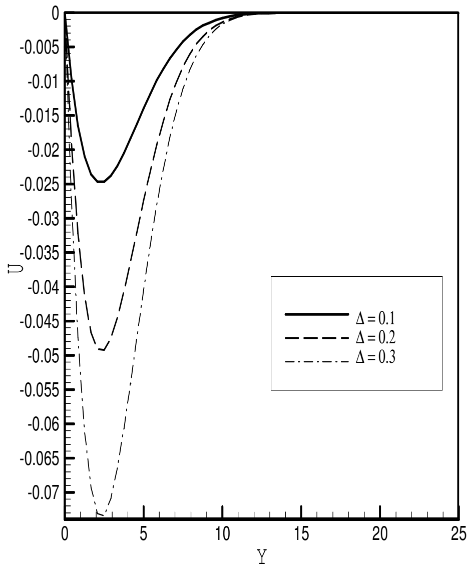

Figure 3. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 10$.

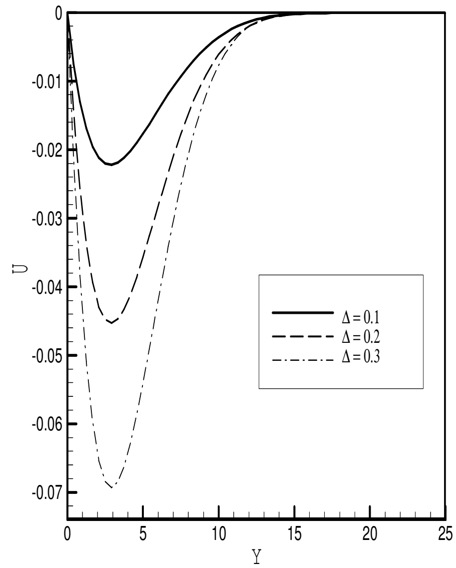

Figure 4. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 20$.

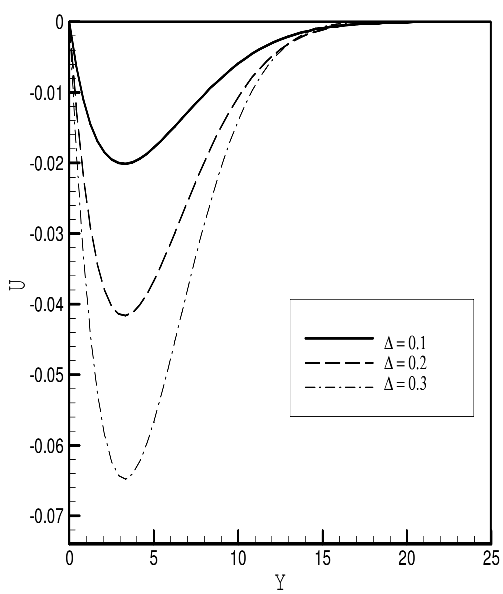

Figure 5. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 30$.

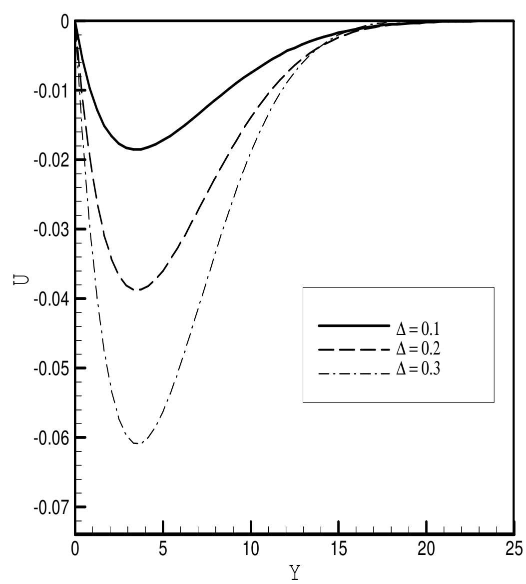

Figure 6. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 40$.

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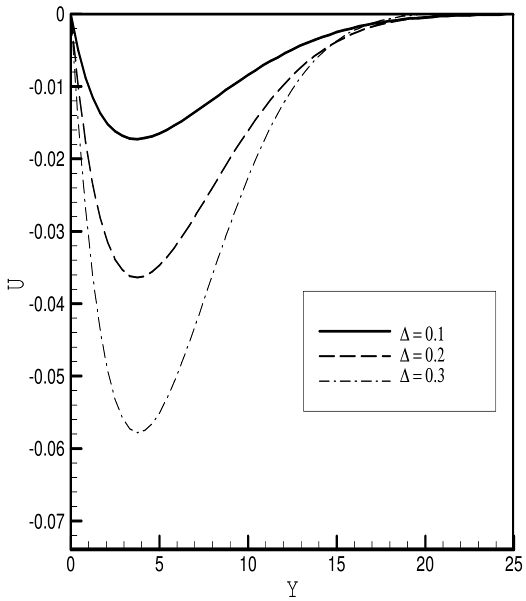

Figure 7. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 50$.

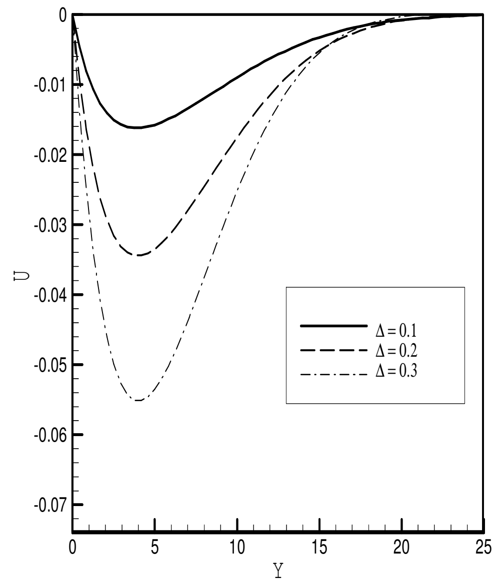

Figure 8. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 60$.

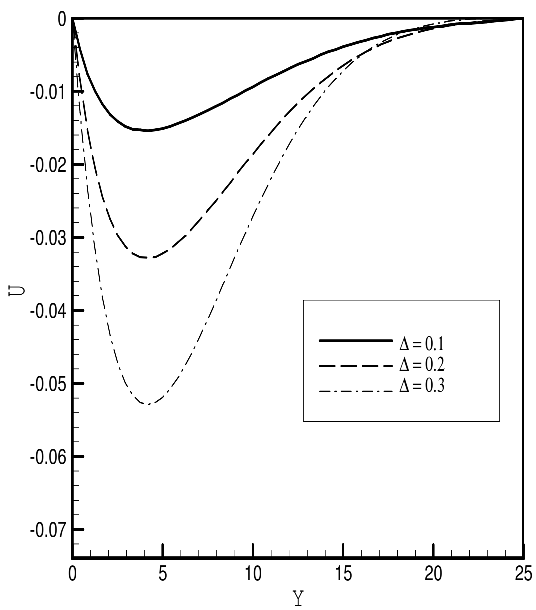

Figure 9. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 70$.

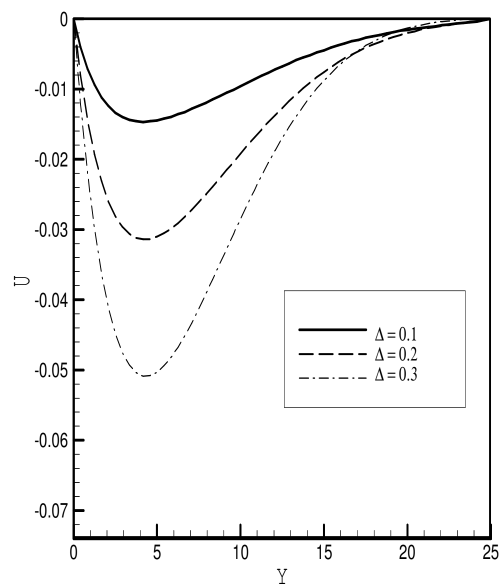

Figure 10. Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 80$.

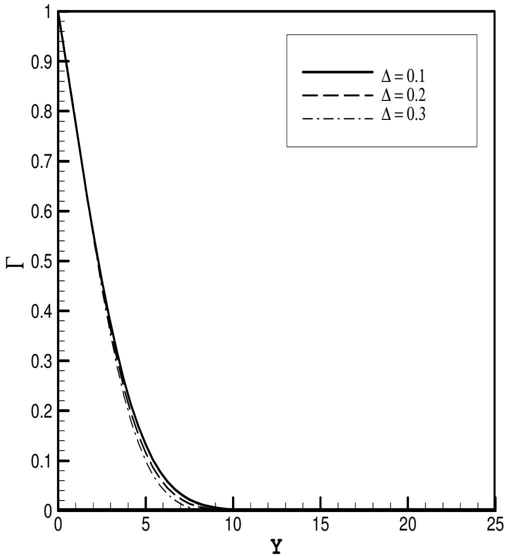

Figure 11. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 10$.

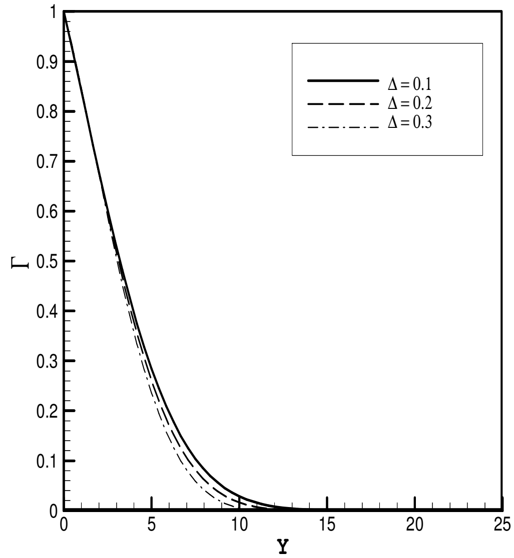

Figure 12. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 20$.

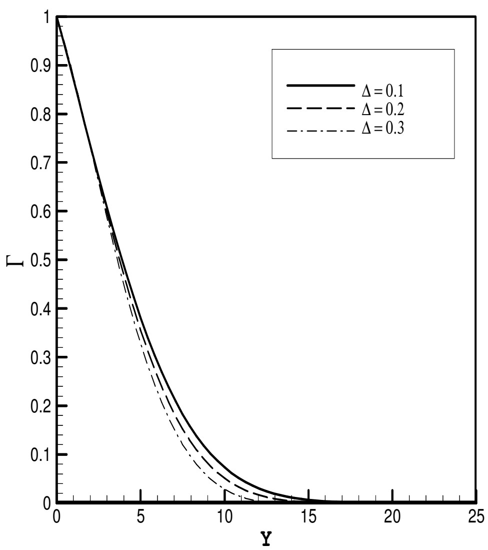

Figure 13. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 30$.

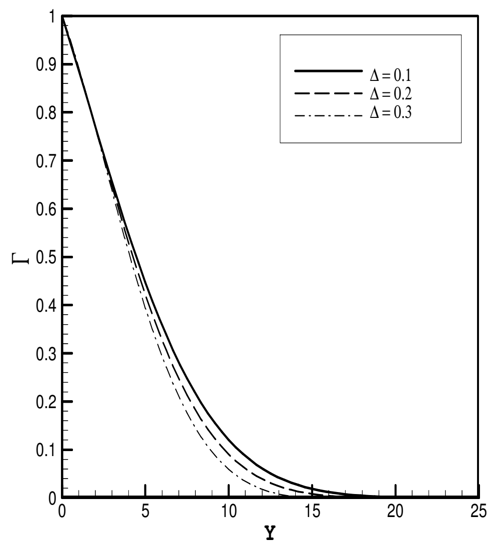

Figure 14. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 40$.

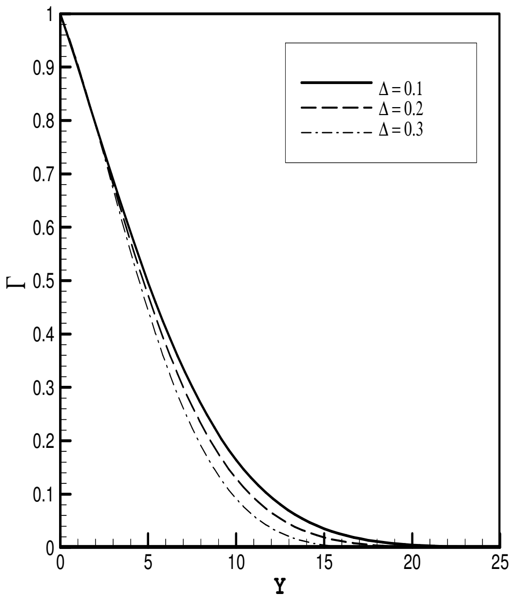

Figure 15. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 50$.

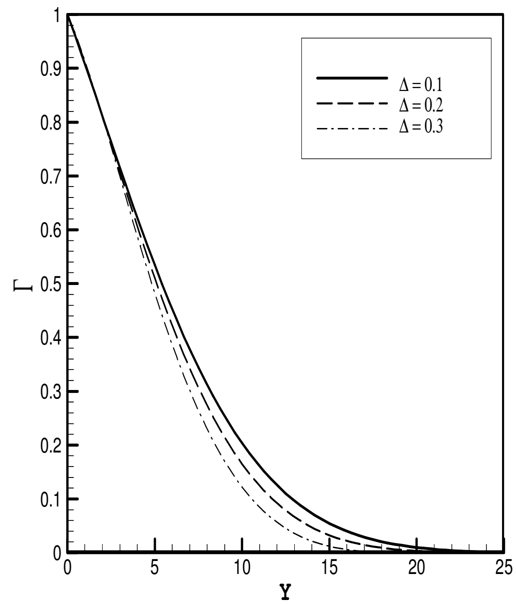

Figure 16. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 60$.

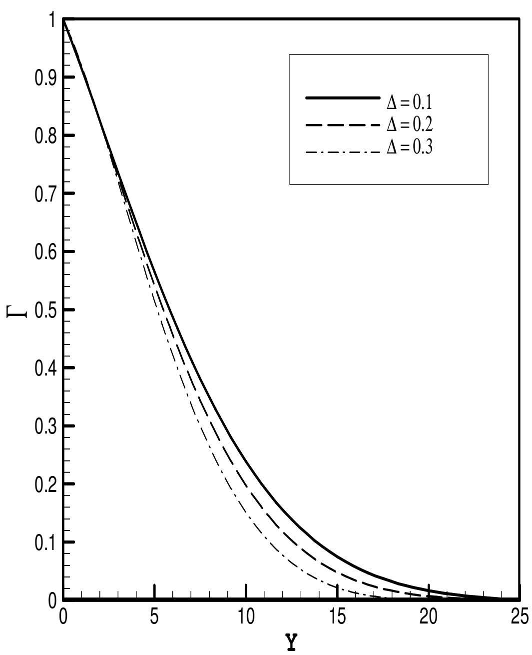

Figure 17. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 70$.

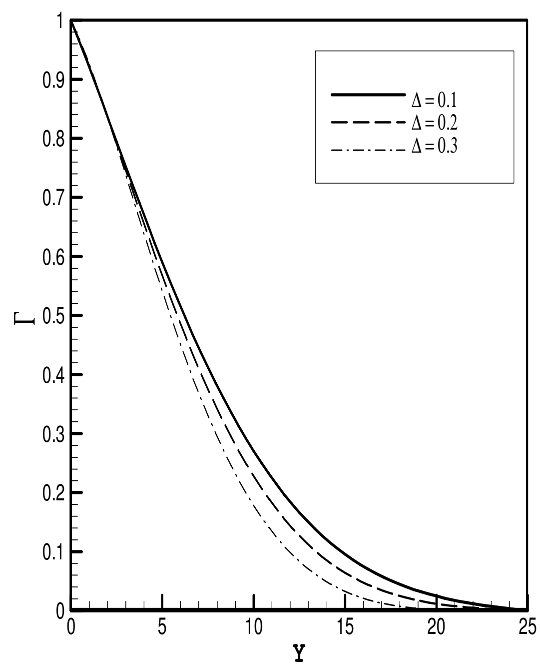

Figure 18. Angular Velocity Profile for different values of Microrotation parameter (Δ) at time $\tau = 80$.

RESULTS AND DISCUSSION

In this study, the effects of MHD unsteady micro-polar fluid behavior through vertical porous plate have been investigated using the finite difference technique. To study the physical situation of this problem, we have computed the numerical values by finite difference technique of velocity and angular velocity at the plate. It can be seen that the solutions are affected by the parameters namely, Micro-rotation parameter (Δ), Spin gradient viscosity parameter (Λ) and the vortex viscosity parameter (λ). The main goal of the computation is to obtain the steady state solutions for the non-dimensional velocity $\boldsymbol{U}$ and micro-rotation Γ' for different values of Micro-rotation parameter (Δ). For this computations the results have been calculated and presented graphically by dimensionless time $\tau = 10$ up to$\tau = 80$. The results of the computations show little changes for $\tau = 10$ to $\tau = 60$. But while arising at $\tau = 70$ and 80 the results remain approximately same. Thus the solution for $\tau = 80$ are become steady-state. Moreover, the steady state solutions for transient values of $\boldsymbol{U}$ and Γ' are shown in figures (3-18), for time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. Figures (3-10) show the velocity profile for different values of Micro-rotation parameter $\Delta = 0.1, 0.2, 0.3$ at time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. From this figures it is observed that the velocity profile decreases with the increase of Micro-rotation parameter (Δ) and the velocity profiles are going downward direction. While arising at $\tau = 70$ and 80 the solutions become steady-state.

Other important effects of micro-rotations are shown in figures (11-18) for different values of Micro-rotation parameter (Δ) at time $\tau = 10, 20, 30, 40, 50, 60, 70, 80$ respectively. It is seen from this figures that the micro-rotation decreases with the increase of micro-rotation parameter (Δ) and is going to the downward direction from the vertical wall with the increase of time. While arising at $\tau = 70$ and 80 the results remain approximately same.

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