INVESTIGATION AT GLOBAL AND MEDITERRANEAN SCALES OF MEAN SEA LEVEL HEIGHT VARIABILITY BY SINGULAR SPECTRUM ANALYSIS

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ABSTRACT

The spatial sampling offered by satellite altimetry and its continuity during the last years are major assets to provide an improved vision of the oceanic variability. In this paper, the seasonal and interannual variability of global and Mediterranean mean sea level height variability are studied through the maps of sea level anomalies (MSLAs) that extend back to 1993. The singular spectrum analysis (SSA) method is applied to the averaged MSLAs time series on global and Mediterranean scale separately. The SSA technique shows that the global mean sea level variability is dominated by an increasing trend, which represents 91.52% of the initial MSLAs time series. The rate of the global long-term trend as seen by SSA appears to be about 2.8 mm yr^{-1}. Also, the SSA shows that the global mean sea level is object of significant harmonic components: annual signal, semi-annual signal and 4 months signal. The amplitude of the grouped seasonal components varies between -0.83 and 1.04 cm. Likewise, the first harmonic components in the Mediterranean sea are: annual signal, semi-annual signal and 7 months signal, where the annual frequency signal is particularly strong. Its contribution represents 72.38 % of the initial averaged MSLAs and its amplitude is of 14.4 cm. The rate of mean sea level height variability over the Mediterranean sea is estimated of 1.72 mm yr^{-1}.

Keywords: mean sea level variability; global and Mediterranean scales; maps of sea level anomalies (MSLAs); singular spectrum analysis; harmonics and trend extraction.

INTRODUCTION

Long-term mean sea level change is a variable of considerable interest in the studies of global climate change. It can provide an important corroboration of predictions by climate models of warming. The purpose of this paper is to give a detailed description of the changes in global and Mediterranean mean sea level variability through the averaged sea level anomalies time series at the two scales by using the SSA technique. Thus, the result of the SSA processing is a decomposition of these two time series into several components, which can be identified as trend, seasonalities and other oscillatory series or noise components.

SSA originated between the late 70s and early 80s. In some cases, the names Caterpillar approach, Principal Component Analysis for time series or Karhunen-Loeve decomposition of time series are also used. SSA technique is a powerful technique of time series analysis incorporating the elements of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing. The possible application areas of SSA are diverse: from mathematics and physics to economics and financial mathematics, from meterology and oceanology to social science and market research. Several book chapters, papers and softwares about SSA technique are available at the SSAwiki website. For variety of the application of SSA see Hassani et al. (2007, 2009 and 2010), Ghodsi et al. (2009, 2010), Hassani and Zhigljavsky (2009), Hassani and Thomakos (2010), Haddad et al. (2010) and references therein.

Note that we are motivated to use SSA because it is a nonparametric technique that works with arbitrary statistical processes, whether linear or nonlinear, stationary or non-stationary, Gaussian or non-Gaussian. Moreover, contrary to the traditional methods of time series analysis, SSA makes no prior assumptions about the data (Hassani, 2010, Hassani and Thomakos, 2010).

The outline of this paper is as follows. Section 2 presents a brief introduction to singular spectrum analysis. Section 3 describes the altimetric datasets used in our study. Section 4 uses the SSA technique to analyze the global and Mediterranean mean sea level height variability. A summary is given in section 5.

DESCRIPTION OF THE SSA

The main idea of SSA is performing a singular value decomposition (SVD) of the trajectory matrix obtained from the original time series with a subsequent reconstruction of the series. The basic version of SSA consists of five steps, which are performed as follows (Golyandina et al., 2001 and Hassani, 2007):

Step 1. (Computing the trajectory matrix): this transfers a one-dimensional time series $Y_T = (y_1, \dots, y_T)$ into the multi-dimensional series $X_1, \dots, X_K$ with vectors $X_i = (y_i, \dots, y_{i+L-1})' \in \mathbf{R}^L$, where $K = T - L + 1$. The single parameter of the embedding is the window length L. The result of this step is the trajectory matrix $X = [X_1, \dots, X_K]$:

$$X = (x_{ij})_{i,j=1}^{L,K} = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_K \\ y_2 & y_3 & y_4 & \cdots & y_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & y_{L+2} & \cdots & y_T \end{bmatrix}$$

The trajectory matrix X is a Hankel matrix, which means that all the elements along the diagonal $i + j = const$ are equal.

Step 2. (Constructing a matrix for applying SVD): compute the matrix XX^T.

Step 3. (SVD of the matrix XX^T): compute the eigenvalues and eigenvectors of the matrix XX^T and represent it in the form $XX^T = P\Lambda P^T$. Here $\Lambda = diag(\lambda_1, \dots, \lambda_L)$ is the diagonal matrix of eigenvalues of $\mathbf{XX}^T$ ordered so that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq 0$ and $P = (P_1, P_2, \dots, P_L)$ is the

corresponding orthogonal matrix of eigen-vectors of .

Step 4. (Selection of eigen-vectors): select a group of $l \ (1 \leq l \leq L)$ eigenvectors $\mathbf{P}_{i_1}, \mathbf{P}_{i_2}, \dots, \mathbf{P}_{i_l}$.

The grouping step corresponds to splitting the elementary matrices $\mathbf{X}_i$ into several groups and summing the matrices within each group. Let $\mathbf{I} = \{\mathbf{i}_1, \dots, \mathbf{i}_l\}$ be a group of indices $\mathbf{i}_1, \dots, \mathbf{i}_l$. Then the matrix $\mathbf{X}_I$ corresponding to the group I is defined as $\mathbf{X}_I = \mathbf{X}_{i_1} + \cdots + \mathbf{X}_{i_l}$.

Step 5. (Reconstruction of the one-dimensional series): compute the matrix $\tilde{X} = \|\tilde{x}_{i,j}\| = \sum_{k=1}^{l} P_{i_k} P_{i_k}^T X$ as an approximation to X. Transition to the one-dimensional series can now be achieved by averaging over the diagonals of the matrix $\tilde{X}$.

Data used

The two datasets distributed by AVISO (Archiving, Validation and Interpretation of Satellite Oceanographic data) are used here:

- the global merged and gridded of Delayed-Time MSLA (Maps of Sea Level Anomaly): These datasets cover all oceans and seas and go from the beginning of 1993 to the end of 2009 with a sampling rate of seven days (one week). The latitude and longitude resolutions of these maps are 1/3 x 1/3 degrees.
- the Mediterranean D-T MSLA: These datasets over the Mediterranean basin (30°N-46°N, 5°W-36°E) go from the beginning of 1993 to the end of 2009, with 1/8 x 1/8 degrees of latitude and longitude resolutions and a sampling rate of seven days (one week).

The main input data for the DT processing are the Geophysical Data Records produced by NASA or CNES (T/P, Jason-1, Jason-2), ESA (ERS-1, ERS-2, ENVISAT), or NOAA (GFO, Geosat) which are therefore of the highest quality, notably in terms of orbit determination. All of the standard corrections to the altimeter data were applied including removal of ocean tides and an inverted barometer correction.

Note that the sea level anomaly (SLA), define as variations of the sea surface height with respect to the mean sea surface, is generally used as precious and main indicator for development of scientific applications which aims to study the ocean variability (mesoscale circulation, seasonal variation, El Niño...).

For each dataset, a temporal series of averaged MSLAs (one mean value per week) is computed on each individual map from an average of all the grid values. The two obtained series with a length of $T = 8\square 7$ are used separately when applying the SSA technique. Figures 1 and 2 represent the temporal series of averaged MSLAs at global scale and in the Mediterranean sea respectively.

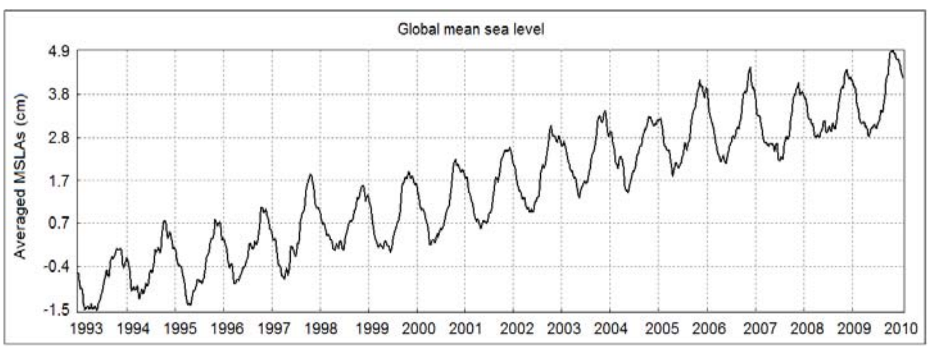

Figure 1: Temporal series of averaged global MSLAs during the period 1993-2009.

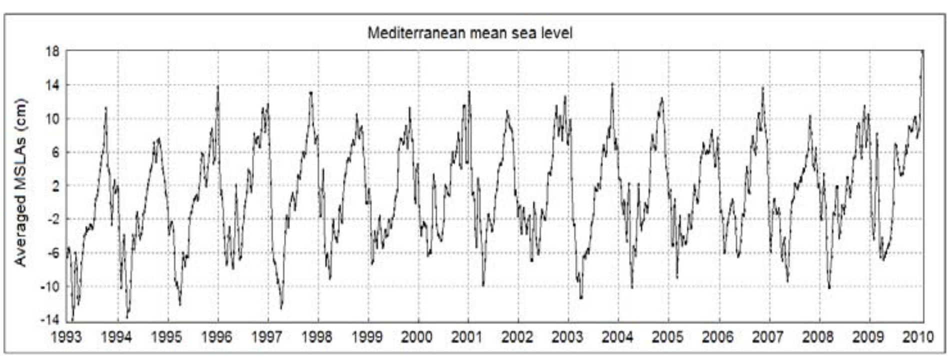

Figure 2: Temporal series of averaged MSLAs in the Mediterranean Sea.

DATA ANALYSIS

In order to extract the mean characteristics of global and Mediterranean sea level height variability, we apply the SSA technique to the two temporal series of averaged MSLAs computed at global and Mediterranean scale respectively.

All of the results and figures in the following two applications are obtained by means of the CaterpillarSSA 3.40 software available at GistaT Group website. This program performs extended analysis, forecasting and change-point detection for one-dimensional time series and analysis/forecast of multi-dimensional time series.

Global mean sea level variability analysis

Decomposition: Window length and SVD

The window length L is the only parameter in the decomposition stage. Selection of the proper window length depends on the problem in hand and on preliminarily information about the time series. Theoretical results tell us that L should be large enough but not greater than $T/2$. Furthermore, if we know that the time series may have a periodic component with an integer period (for example, if this component is a seasonal component), then to get better separability of this periodic component it is advisable to take the window length proportional to that period (Hassani, 2010). Using these recommendations, we take $L = 108$. So, based on this window length and on the SVD of the trajectory matrix $(108\,\mathbf{x}\,1\square8)$, we have 108 eigentriples. Figure 3 represents the plot of logarithms of the 108 singular values.

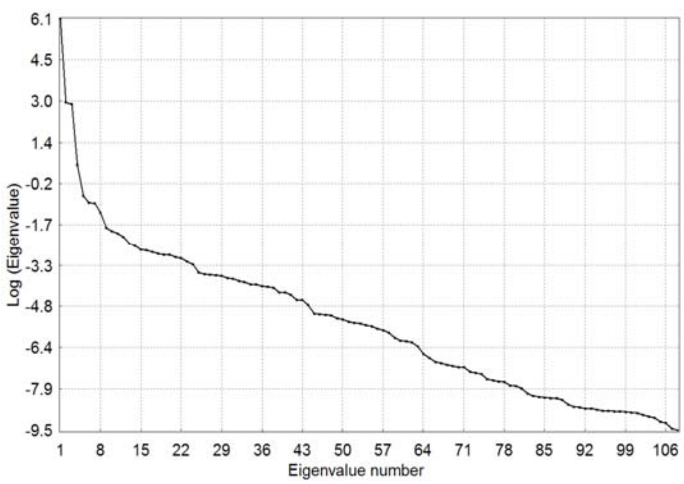

Figure 3: Logarithms of the 108 eigenvalues.

Harmonic components identification

A natural hint for grouping is the matrix of the absolute values of the w-correlations, corresponding to the full decomposition (in this decomposition each group corresponds to only one matrix component of the SVD). If two reconstructed components have zero w-correlation it means that these two components are separable. Large values of w-correlations between reconstructed components indicate that they possibly should be gathered into one group and correspond to the same component in SSA decomposition (Hassani, 2010). Figure 4 shows the w-correlations for the 108 reconstructed components in a 20-grade grey scale from white to black corresponding to the absolute values of correlations from 0 to 1. Here zero w-correlation values occur around component 15. Based on this information, we select the first 15 eigentriples for the identification of the harmonic components and consider the rest as noise.

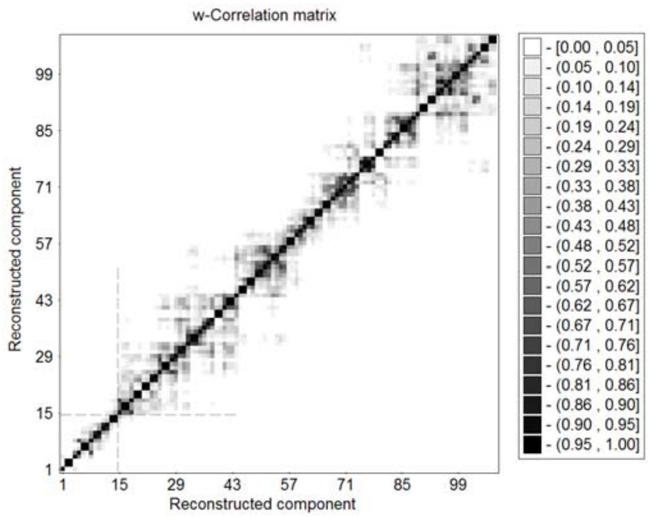

Figure 4: Matrix of w-correlations for the 108 reconstructed components.

In practice, the singular values of the two eigentriples of a harmonic series are often very close to each other, and this fact simplifies the visual identification of the harmonic components. Therefore, explicit plateau in the eigenvalue spectra prompts the ordinal numbers of the paired eigentriples of harmonic components. Another way to identify the harmonic components of the series is to examine the pairwise scatterplots of the singular vectors. Pairwise scatterplots like spiral circles, spiral regular polygons or stars determine periodic components of the time series provided these components are separable from the residual component. Figure 5 and 6 represent the first 16 pairwise scatterplots and the first 16 principal components respectively, ordered by their contribution in the decomposition.

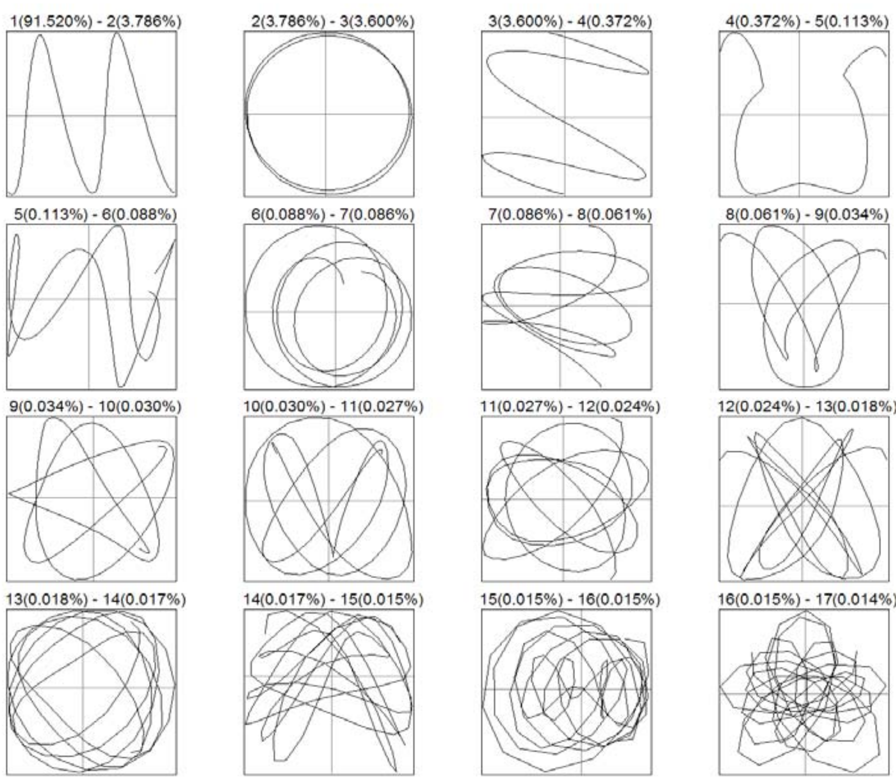

Figure 5: Scatterplots of the first 16 paired eigenvectors and its contribution.

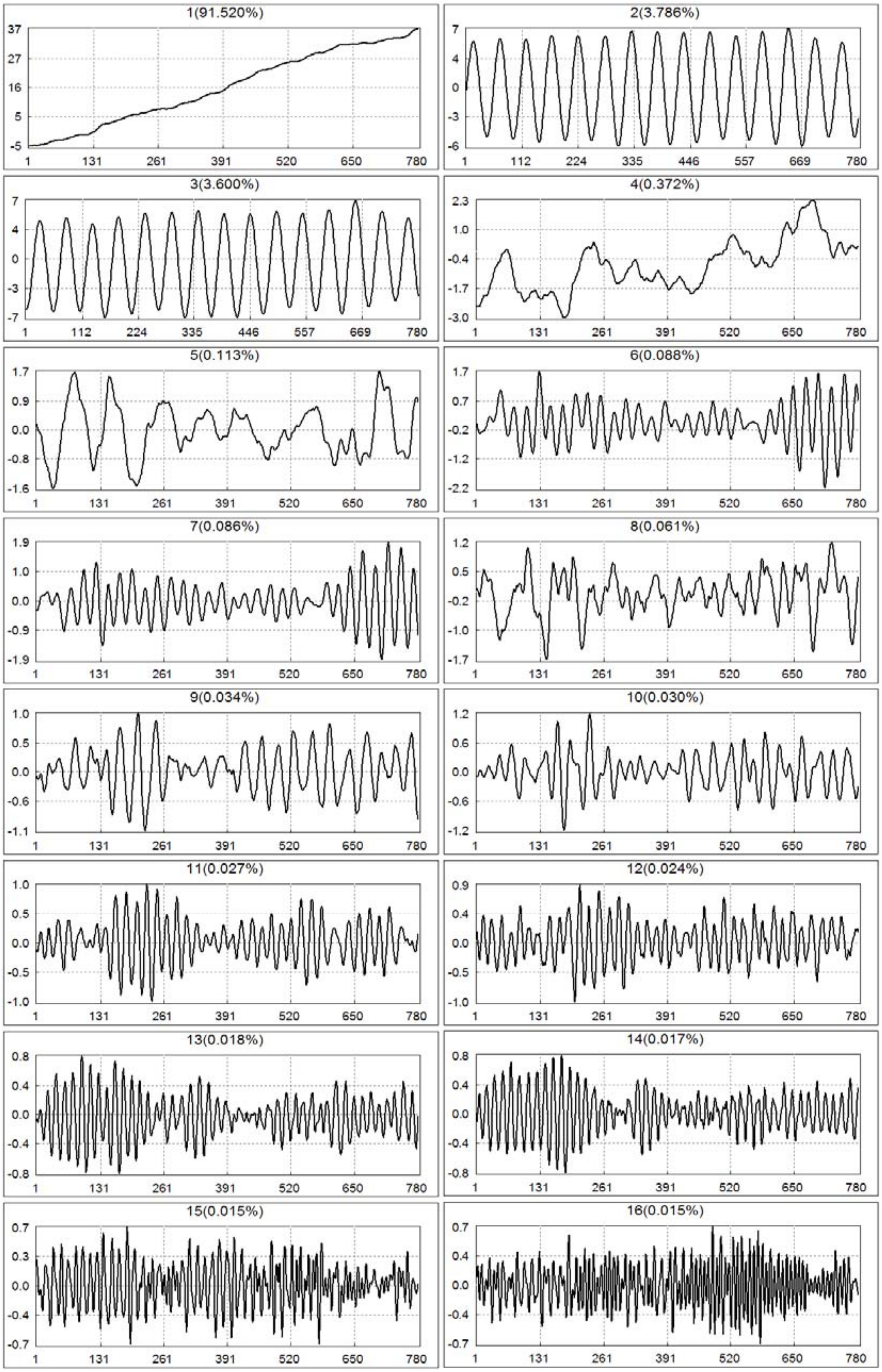

Figure 6: Principal components related to the first 16 eigentriples and its contribution.

It can be seen from Figure 3 and 5, that there are three evident pairs with almost equal leading singular values, correspond to three harmonic components in the averaged MSLAs series: eigentriple pairs 2-3, 6-7 and 13-14. The contribution these paired harmonic in the original averaged MSLAs temporal series are: 2(3.786%)-3(3.600%), 6(0.088%)-7(0.086%) and 13(0.018%)-14(0.017%) respectively.

Figure 7 represents the periodograms of initial averaged MSLAs series and identified harmonic components (eigentriple pairs 2-3, 6-7 and 13-14). It can be seen from the periodogram analysis that the frequencies of the 2-3, 6-7 and 13-14 components agree well with the frequencies of the initial averaged MSLA series. The periodicities of these three identified harmonics are of 52.69 weeks (annual signal), 27.05 weeks (semi-annual signal) and 16.97 weeks (four months signal) respectively.

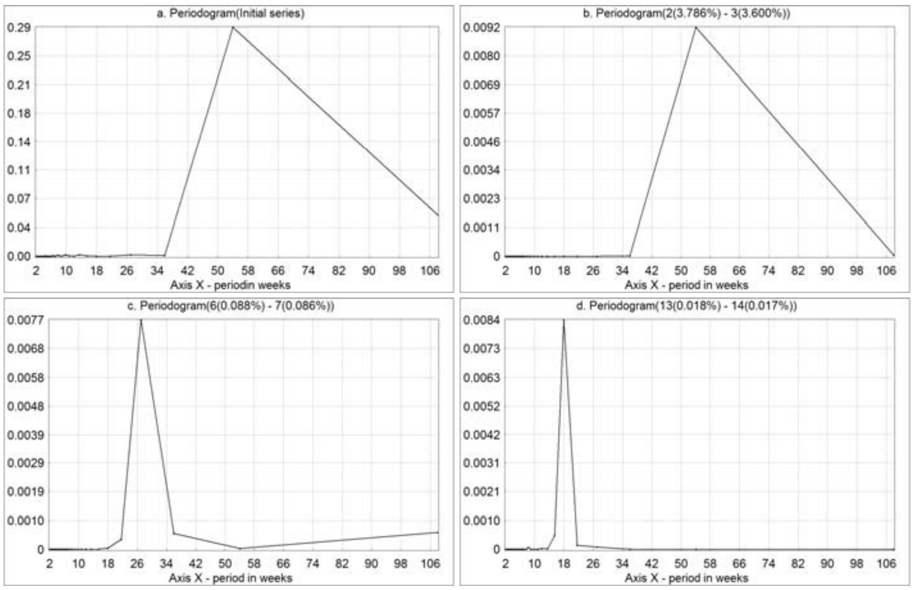

Figure 7: Periodograms: (a) initial MSLA serie, (b) eigentriple pair 2-3, (c) eigentriple pair 6-7, (d) eigentriple pair 13-14.

Trend identification

Assume that the time series itself is such a component alone. Practice shows that in this case, one or more of the leading eigenvectors will be slowly varying as well. We know that eigenvectors have (in general) the same form as the corresponding components of the initial time series. Thus we should find slowly varying eigenvectors. It can be done by considering one-dimensional plots of the eigenvectors. It can be seen from Figure 6, that the trend is obtained from the first eigentriple which represents a contribution of 91.52 % of the initial averaged MSLAs series.

Reconstruction and residuals series

The SSA allows assessing the noise affecting the time series by extracting the trend and identified seasonal components from the initial series. Therefore, we select the 1, 2, 3, 6, 7, 13 and 14 eigentriples for the reconstruction of the initial averaged MSLAs series and consider the rest as noise (residuals series). Figure 8.a shows the extracted trend (represented in solid line) which is obtained from the first eigentriple and the initial averaged MSLAs series (represented in dashed line). The reconstructed trend clearly follows the main tendency in the averaged MSLAs series. It can be seen that the trend of global mean sea level is subject to significant rise, from -0.5 to 4 cm during the period 1993-2009. The trend exhibits a linear slope of 2.80 mm yr–1. Figure 8.b represents the grouping of the three harmonic components (eigentriple pairs 2-3, 6-7 and 13-14) and clearly shows the same pattern of seasonality as in the initial series. The amplitude of the grouped seasonal components are between -0.83 and 1.04 cm. The dotted and the solid line in Figure 8.c correspond to the initial series and the reconstructed series from the trend and the three harmonic components. Figure 8.d shows the residuals obtained from grouping of the eigentriples, which do not contain elements of trend and oscillations. The residuals represent 0.88% of the initial series, and its amplitude is about 0.8 cm.

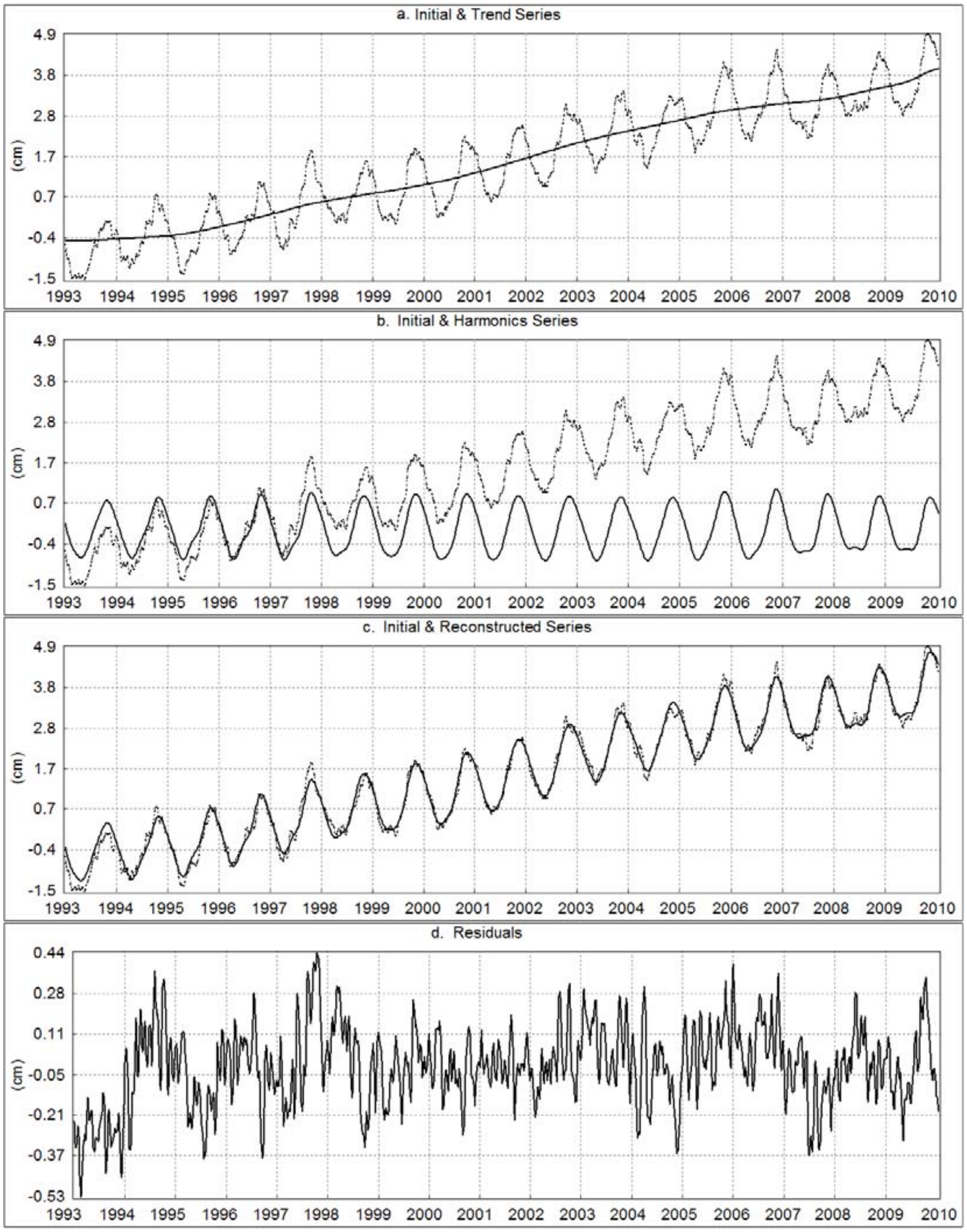

Figure 8: Reconstruction: (a) trend, (b) harmonics, (c) reconstructed series, (d) noise.

Mediterranean mean sea level variability analysis

Here, we take the window length $L - 4\square4$. So, based on this window length and on the SVD of the trajectory matrix $(444 \times 4\square4)$, we have 444 eigentriples. Figure 9 shows the w-correlations for the 444 reconstructed components in a 20-grade grey scale. Here zero w-correlation values occur around component 20. Based on this information, we select the first 20 eigentriples for the identification of the harmonic components. Figure 10 represents the plot of logarithms of the first 20 singular values.

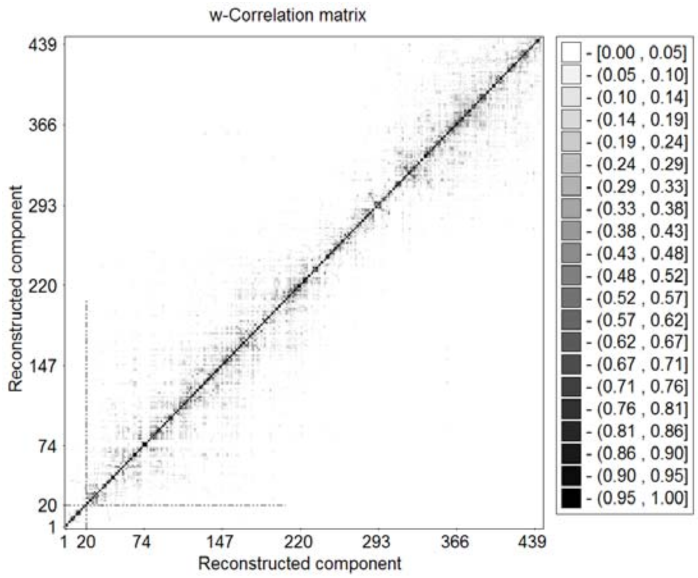

Figure 9: Matrix of w-correlations for the 444 reconstructed components.

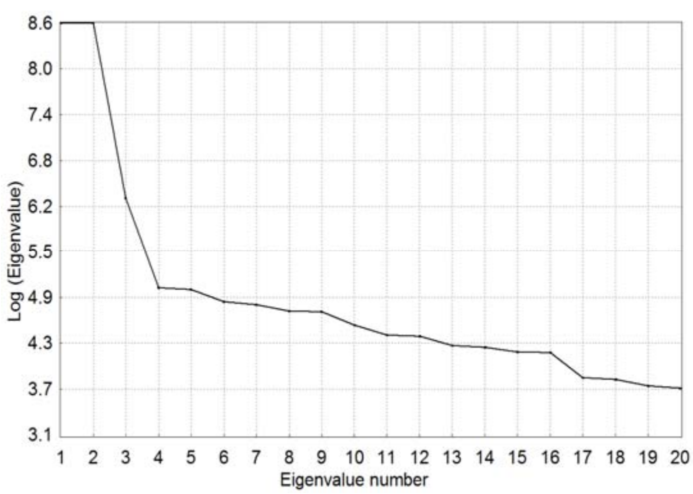

Figure 10: Logarithms of the first 20 eigenvalues.

It can be seen from Figures 4 that there are several paired harmonic eigenvectors. The first three evident paired harmonic eigenvectors are 1-2, 4-5 and 8-9. The contribution these paired harmonic in the original averaged MSLAs temporal series over Mediterranean sea are: 1(36.205%)-2(36.171%), 4(1.058%)-5(1.034%) and 8(0.772%)-9(0.772%) respectively. The periodogram analysis indicates that the periodicities of the three first identified paired harmonic eigenvectors (1-2, 4-5, and 8-9) are: 51.99 weeks, 26.00 weeks and 31.63 weeks respectively.

A model for the Mediterranean Sea level variability is computed using the three identified spectral components by the SSA. Therefore, we select the 1-2, 4-5 and 8-9 eigentriples for the reconstruction of the initial averaged MSLAs series and consider the rest as noise (residuals series). The grouped 1-2, 4-5 and 8-9 eigentriples represents 76.012% of the original series.

The long-term trend is estimated by least-squares fitting of the residuals after elimination of this model. The global analysis shows a trend in the Mediterranean Sea of 1.72 mm yr^{-1}.

Figures 11.a, 11.b and 11.c represent the three reconstructed harmonic components of the MSLAs series, given by the eigentriple pairs 1-2, 4-5 and 8-9 respectively. The amplitudes of these components appear to be about: 14.4 cm, 3.8 cm and 1.96 cm.

The dotted and the solid line in Figure 12.a correspond to the original series and the reconstructed series from the three harmonic components (1-2, 4-5 and 8-9 eigentriples). The reconstructed series clearly shows the same pattern of seasonality as in the initial averaged MSLA series. Figure 12.b shows the residuals used for trend estimation.

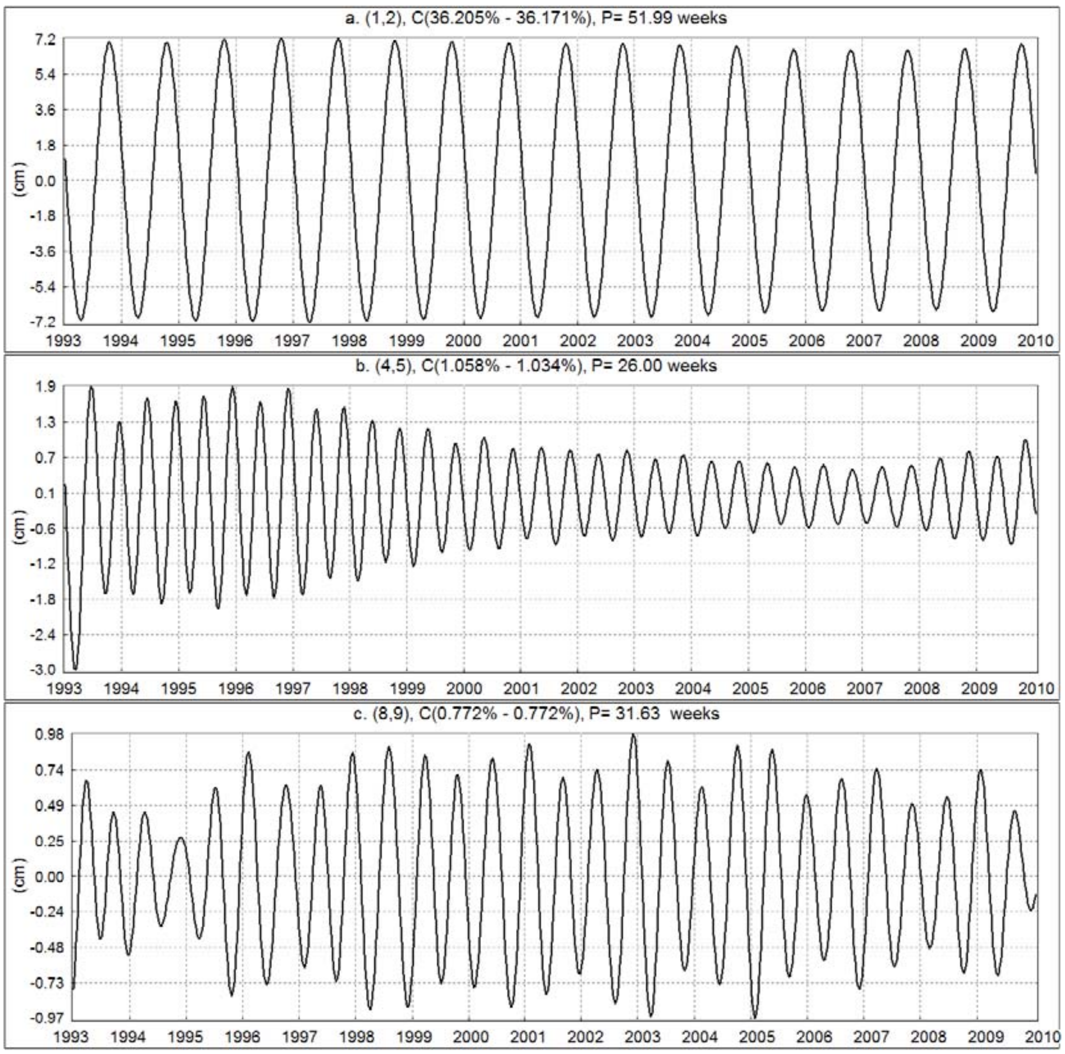

Figure 11: Reconstruction of the three identified harmonics.

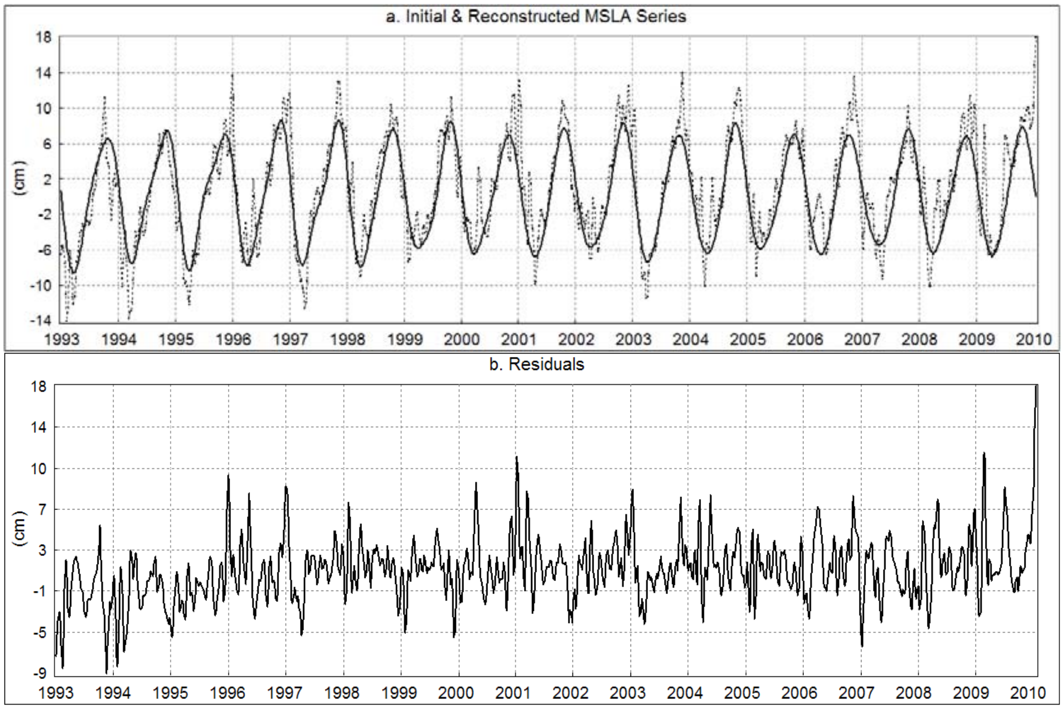

Figure 12: Reconstruction: (a) harmonics, (b) noise.

CONCLUSION

By considering a temporal series of averaged maps of sea level anomalies (MSLAs) at global scale (1993-2009) issue from altimetry satellites, SSA shows that the global mean sea level variability is dominated by an increasing trend which represents 91.52% of the initial time series and by three harmonics: annual signal (a periodicity of 52.69 weeks), semi-annual signal (a periodicity of 27.05 weeks) and four months signal (a periodicity of 16.97 weeks). The contribution of these three harmonics in the initial time series appears to be about 7.39%, 0.18% and 0.04% respectively. The amplitude of the grouped harmonic components fluctuate between -0.83 and 1.04 cm.

The rate of global mean sea level rise, as seen by SSA, is estimated as 2.8 mm yr^{-1} during the last seventeen years (1993 to 2009). This result agrees with the recent study of Aviso Altimetry (2010), which suggested a rate of sea level rise of 2.92 mm yr^{-1} over 1993 to mid-2009 (no glacial isostatic adjustment correction). If present trend continue, it will dramatically impact the land, flora, fauna, and people activities established along the coastlines.

Moreover, the analysis by the SSA technique of the temporal series of averaged MSLAs over Mediterranean sea (1993-2009) shows that the Mediterranean mean sea level variability is dominated by several harmonics. The first three harmonics are: annual signal (a periodicity of 51.99 weeks), semi-annual signal (a periodicity of 26.00 weeks) and seven months signal (a periodicity of 31.63 weeks). The annual frequency signal is particularly strong in the Mediterranean sea. Its contribution represents 72.38 % of the initial MSLAs time series, while its amplitude is of 14.4 cm.

A model for the Mediterranean Sea level variability, computed using the first three identified spectral components, exhibits an amplitude of 17 cm. The long-term trend, estimated after elimination of the three periodic components identified by the SSA, show that the Mediterranean Sea level has been increasing 1.72 millimeters per year since 1993.

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