

PRESERVICE SECONDARY SCHOOL MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE OF CALCULATING PERIMETER AND AREA

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ABSTRACT

This purpose of this article is to examine preservice secondary school mathematics teachers' subject matter knowledge of calculating perimeter and area of composite figure. Clinical interview technique was employed to collect the data. Interview sessions were recorded using digital video camera and tape recorder. Subjects of this study consisted of eight preservice secondary school mathematics teachers enrolled in a Mathematics Teaching Methods course at a public university in Peninsula Malaysia. They were selected based on their majors (mathematics, biology, chemistry, physics) and minors (mathematics, biology, chemistry, physics). This article presents the analysis of the responses of the subjects related to a particular task. The finding suggests that most of the preservice secondary school mathematics teachers in this study had adequate procedural knowledge of calculating perimeter and area of composite figure. All the preservice teachers in this study understand the general measurement convention that perimeter and area is measured by linear units and square units respectively. All of them wrote the measurement units (without being probed) for the answers of the perimeter and area of the composite figure that they have calculated. Nevertheless, none of the preservice teachers in this study check the correctness of their answers. Once getting an answer, they seemed to satisfy that the task was finished. When probed to check answer, then only they suggested the strategies that they would use to check the answers. The preservice teachers in this study employed two types of strategies to verify their answers for calculating perimeter and area of composite figure, namely recalculating strategy and alternative method. The similarities and differences of the findings of this study in comparison with the findings of previous studies were discussed.

Keywords: preservice secondary school mathematics teachers, subject matter knowledge, perimeter and area, case study, clinical interview.

INTRODUCTION

One cannot teach what one does not know. Teachers must have in-depth knowledge of mathematics they are going to teach. Therefore, it is important that a teacher need to have a comprehensive knowledge of mathematics to enable him or her to organize teaching so that students can learn mathematics meaningfully. Fennema and Franke (1992) advocated that "no one questions the idea that what a teacher know is one of the most important influences on what is done in classroom and ultimately on what students learn" (p. 147). Furthermore, "teachers who do not themselves know a subject well are not likely to help students learn this content." (Ball, Thames, & Phelps, 2008, p. 404). This applies also to mathematics teacher.

One of the learning outcomes enlisted in the Form One Mathematics Curriculum Specifications is 'find the areas of composite figures made up of rectangle, parallelogram, triangle, or trapezium' (Ministry of Education Malaysia, 2003). This learning outcome could be extended to perimeter as well. However, previous study (Baturu & Nason, 1996) revealed that preservice primary school teachers in their study had inadequate procedural knowledge of calculating area of the given shapes.

Moreover, Cavanagh (2008) and van de Walle (2007) found that students tend to confuse with the slanted side (slanted height) and the height (perpendicular height) of a parallelogram.

It is a general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as mm^2 , cm^2 , m^2 , km^2) respectively. However, Tierney, Boyd, and Davis (1990) noticed that many prospective primary school teachers from a teachers college in their study labelled the area measurements in linear units. Likewise, Baturo and Nason (1996) found that several preservice primary school teachers in their study wrote the calculated area measurement in linear unit such as 128 cm. Similarly, Cavanagh (2008) revealed that high school students in his study inappropriately labelled the length of sides in cm^2 or areas in cm. They did not understand the general measurement convention that length is measured in linear units while area is measured in square units.

Baturo and Nason (1996) revealed that most preservice primary school teachers in their study who attempted to verify their answers did so by recalculating strategy or using the inverse operation. They never think of using an alternative method to verify their answers. Mosenthal and Ball (1992) argued that assessing the reasonableness of one's solutions is a hallmark of understanding. Moreover, checking the correctness or reasonableness of one's answers or solutions is a good behavior in mathematics (Nik Azis, 2007). However, Baturo and Nason (1996) found that majority of the preservice primary school teachers in their study had to be prodded towards checking their answers. Once getting an answer, they seemed to satisfy that the task was finished.

PURPOSE OF THE STUDY

The purpose of this study was to examine preservice secondary school mathematics teachers (PSSMTs)' subject matter knowledge of calculating perimeter and area of composite figure. Specifically, this study aimed to investigate the preservice teachers' procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge of calculating perimeter and area of composite figure.

CONCEPTUAL FRAMEWORK OF THE STUDY

Nik Azis (1996) suggested that there are five basic types of knowledge, namely conceptual knowledge, procedural knowledge, linguistic knowledge, strategic knowledge, and ethical knowledge. This applies also to subject matter knowledge. In the present study, the researchers have adapted Nik Azis's (1996) categorization of knowledge to examine the PSSMTs' subject matter knowledge of calculating perimeter and area of composite figure.

METHODOLOGY

Research Design

In this study, the researchers employed case study research design to examine, in-depth, PSSMTs' subject matter knowledge of calculating perimeter and area of composite figure. "A case study design is used to gain an in-depth understanding of the situation and meaning for those involved" (Merriam, 1998, p. 19). Several researchers (e.g., Aida Suraya, 1996; Chew, 2007; Lim, 2007; Rokiah, 1998; Seow, 1989; Sharifah Norul Akmar, 1997; Sutriyono, 1997) employed case study research design to study Malaysian students, preservice teachers, and lecturers.

Selection of Subjects

The researchers employed purposeful sampling to select the subjects (sample) for this study. Eight subjects (sample) were selected for the purpose of this study. They were PSSMTs from a public university in Peninsula Malaysia enrolled in a 4-year Bachelor of Science with Education (B.Sc.Ed.) program, majored or minored in mathematics. These subjects enrolled for a one-semester mathematics teaching methods course during the data collection of this study. The mathematics teaching methods

course was offered to B.Sc.Ed. program students who intended to major or minor in mathematics. The researchers had selected four B.Sc.Ed. program students who majored in mathematics, and four B.Sc.Ed. program students who minored in mathematics for the purpose of this study. Each PSSMT was given a pseudonym, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, in order to protect the anonymity of all interviewees. The brief background information about the subjects is shown in Table 1.

Table 1. Subjects' Ethnicity, Gender, Age, Major, Minor, and CGPA

Subject	Ethnicity	Gender	Age	Major	Minor	CGPA
Usha	Indian	Female	(21, 9)	Mathematics	Biology	2.92
Mazlan	Malay	Male	(21, 8)	Mathematics	Chemistry	2.70
Patrick	Bidayuh	Male	(21, 7)	Mathematics	Chemistry	3.04
Beng	Chinese	Female	(22, 9)	Mathematics	Physics	3.82
Roslina	Malay	Female	(21, 8)	Biology	Mathematics	3.15
Liana	Malay	Female	(21, 5)	Chemistry	Mathematics	2.77
Tan	Chinese	Male	(22, 7)	Chemistry	Mathematics	3.69
Suhana	Malay	Female	(20, 10)	Physics	Mathematics	2.52

Instrumentation

Eight types of interview tasks were devised for this study. This paper reports only the responses of the subjects on Task 6.1 (see Appendix A). In Task 6.1, subjects were required to help his or her student to calculate the perimeter and area of the given diagram (Diagram 1) that involved composite figure, namely rectangle and parallelogram/triangle. The objective of this task was to determine the subjects' procedural knowledge for calculating perimeter and area for the composite figure. Task 6.1 was used to determine the subjects' linguistic knowledge of standard units of perimeter and area measurement by determining whether the subject use the correct standard units of measurement for perimeter (cm) and area (cm²) when they write the answers for these measurements. Task 6.1 was also used to determine the subjects' strategic knowledge for checking the correctness of their answers for perimeter as well as area. Task 6.1 was used to determine the subjects' ethical knowledge by ascertaining whether the subjects write units of measurement upon completing a task. This task was also used to ascertain whether the subjects check the correctness of their answers.

Data Collection

Data for this study was collected using clinical interview technique. In the present study, materials collected for analysis consisted of audiotapes and videotapes of clinical interviews, subject's notes and drawings, and researcher's notes during the interviews. The audiotapes and videotapes were verbatim transcribed into written form and the transcriptions were the raw data for this study. The transcriptions include verbal and nonverbal interaction between researcher and subject.

Data Analysis

The data analysis process encompassed four levels. At level one, the audio and video recording of the clinical interview were verbatim transcribed into written form. The transcription included the interaction between the researcher and the subject during the interviews as well as the subject's

nonverbal behaviors. At level two, raw data in the forms of transcription were coded, categorized, and analyzed according to specific themes to produce protocol related to the description of subjects' subject matter knowledge of calculating perimeter and area for the composite figure. At level three, case study for each subject was constructed based on information from the written protocol. At this level, analysis was carried out to describe each subject's behaviors in solving the task. At level four, cross-case analysis was conducted. The analysis aimed to identify pattern of responses of subject matter knowledge of calculating perimeter and area for the composite figure held by the subjects. Based on this pattern of responses, preservice teachers' subject matter knowledge of calculating perimeter and area for the composite figure were summarized.

FINDINGS OF THE STUDY

Procedural Knowledge

Finding of the study suggests that seven of the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, and Usha, have successfully calculated the perimeter of Diagram 1 as 104 cm. Table 2 reveals PSSMTs who have successfully and unsuccessfully calculated the perimeter of Diagram 1.

Table 2. PSSMTs who Have Successfully and Unsuccessfully Calculated the Perimeter of Diagram 1

Calculating the perimeter of Diagram 1	PSSMTs
Successful	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
Unsuccessful	Tan

Of the seven PSSMTs who have successfully calculated the perimeter of Diagram 1, five of them, namely Beng, Mazlan, Patrick, Roslina and Usha, used the list all-and-sum algorithm to calculate the perimeter of the diagram. They listed all the length of sides that surrounded the diagram and then summed them up to get the perimeter of the diagram as 104 cm. Table 3 exhibits the algorithms used by PSSMTs to calculate the perimeter of Diagram 1. The other two PSSMTs, namely Liana and Suhana, used the doubling-and-sum algorithm to calculate the perimeter of the diagram. They doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm.

Table 3. The Algorithms Used by PSSMTs to Calculate the Perimeter of Diagram 1

Algorithms used to calculate the perimeter of Diagram 1	PSSMTs
List all-and-sum	Beng, Mazlan, Patrick, Roslina, Tan, Usha
Doubling-and-sum	Liana, Suhana

Only one PSSMT, namely Tan, have unsuccessfully calculated the perimeter of Diagram 1. Tan mentally cut the triangle TRS of Diagram 1 and pasted it next to the triangle TQR of Diagram 1 so that it formed a rectangle ("TQSR") with the dimension of 15 cm by 8 cm. He used the list all-and-sum algorithm to calculate the perimeter of the diagram, He listed all the length of sides that surrounded the "long" rectangle and then summed them up to get the perimeter of the diagram as 86 cm (the correct answer should be 104 cm). Tan did not know that the "cut and paste" transformation does not conserve the perimeter of a diagram. Thus, he incorrectly calculated the perimeter of the diagram as 86 cm based on the length of sides that surrounded the "long" rectangle formed ($20 + 8 + 15 + 20 + 8 + 15$) and not based on the length of sides that surrounded Diagram 1 ($20 + 17 + 15 + 20 + 17 + 15 = 104$).

Finding of the study suggests that six of the PSSMTs, namely Beng, Liana, Patrick, Roslina, Suhana, and Tan, have successfully calculated the area of Diagram 1 as 420 cm^2 . Table 4 shows PSSMTs who have successfully and unsuccessfully calculated the area of Diagram 1.

Table 4. PSSMTs who Have Successfully and Unsuccessfully Calculated the Area of Diagram 1

Calculating the area of Diagram 1	PSSMTs
Successful	Beng, Liana, Patrick, Roslina, Suhana, Tan
Unsuccessful	Mazlan, Usha

Of the six PSSMTs who have successfully calculated the area of Diagram 1, five of them, namely Beng, Liana, Patrick, Roslina, and Suhana, used the partition-and-sum algorithm to calculate the area of the diagram. They partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Beng, Liana, Patrick, Roslina, and Suhana calculated the areas of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm^2 . Table 5 depicts the algorithms used by PSSMTs to calculate the area of Diagram 1. The other PSSMT, namely Tan, used the “cut and paste” transformation to transform Diagram 1 into a “long” rectangle. He calculated the area of Diagram 1 as the area of the “long” rectangle formed using the area formula of a rectangle where its length and width is 28 cm and 15 cm respectively. Tan got the area of the diagram as 420 cm^2 .

Table 5. The Algorithms Used by PSSMTs to Calculate the Area of Diagram 1

Algorithms used to calculate the area of Diagram 1	PSSMTs
Partition-and-sum algorithm	Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Usha
“Cut and paste” transformation	Tan

The remaining two PSSMTs, namely Mazlan and Usha, have unsuccessfully calculated the area of Diagram 1. They used the partition-and-sum algorithm to calculate the area of the diagram. Mazlan partitioned Diagram 1 into a rectangle PQTU and two triangles, namely QRT and RST while Usha partitioned Diagram 1 into a rectangle PQTU and a parallelogram QRST. They correctly calculated the area of the rectangle as 300 cm^2 . Mazlan viewed the two triangles as parallelogram QRST. Nevertheless, Mazlan and Usha confused with the slanted side and the height of the parallelogram that they used the slanted side QR as the height ($TR = 8 \text{ cm}$) of the parallelogram. Thus, Mazlan and Usha incorrectly calculated the area of the parallelogram as ' $17 \times 15 = 255 \text{ cm}^2$ ' (The area of the parallelogram should be ' $15 \times 8 = 120 \text{ cm}^2$ '). Consequently, they got the area of the diagram as 555 cm^2 (The correct answer should be 420 cm^2 , not 555 cm^2).

Linguistic Knowledge

Finding of the study suggests that all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, used the correct standard unit of measurement for perimeter, namely cm, when they wrote the answer for this measurement of Diagram 1. It indicated that they understand the general measurement convention that perimeter is measured by linear unit. Finding of the study also suggests that all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, used the correct standard unit of measurement for area, namely cm^2 , when they wrote the answer for this measurement of Diagram 1. It indicated that they understand the general measurement convention that area is measured by square unit.

Strategic Knowledge

When probed to check the answer for the perimeter of Diagram 1, seven of the PSSMTs, namely Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, suggested that they would use the recalculating strategy to verify the answer. Beng, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha suggested that they would check the answer for perimeter by recalculating strategy that using the same method and calculate again. Table 6 exhibits the strategies suggested by PSSMTs to check the answer for the perimeter of Diagram 1.

Table 6. Strategies Suggested by PSSMTs to Check the Answer for the Perimeter of Diagram 1

Strategies suggested to check the answer for the perimeter of Diagram 1	PSSMTs
Recalculating strategy	Beng, Mazlan, Patrick, Roslina, Suhana, Tan, Usha
Alternative method	Liana

When probed to check the answer for the perimeter of Diagram 1, Liana suggested that she would use an alternative method to verify the answer. Liana used the doubling-and-sum algorithm to calculate the perimeter of the diagram. She doubled the length of sides UP, PQ, and QR and then summed them up to get the perimeter of the diagram as 104 cm. Liana suggested that she would check the answer for perimeter by using an alternative method, namely list all-and-sum strategy. Liana explained that she could just “15 plus 20 plus 17 plus 15 plus 17 plus 20” (Liana/L991-992) to check the answer for the perimeter.

When probed to check the answer for the area of Diagram 1, half of the PSSMTs, namely Mazlan, Roslina, Suhana, and Usha, suggested that they would use the recalculating strategy to verify the answer. Mazlan, Roslina, Suhana, and Usha suggested that they would check the answer for the area by the recalculating strategy that using the same method and calculate again. Table 7 shows the strategies suggested by PSSMTs to check the answer for the area of Diagram 1.

Table 7. Strategies Suggested by PSSMTs to Check the Answer for the Area of Diagram 1

Strategies suggested to check the answer for the area of Diagram 1	PSSMTs
Recalculating strategy	Mazlan, Roslina, Suhana, Usha
Alternative method	Beng, Liana, Patrick, Tan

When probed to check the answer for the area of Diagram 1, the other half of the PSSMTs, namely Beng, Liana, Patrick, and Tan, used an alternative procedure (alternative method) to generate an answer which could be used to verify their original answer. Beng, Liana, and Patrick used the partition-and-sum algorithm to calculate the area of the diagram. They partitioned Diagram 1 into a rectangle PQTU (labelled as A) and two triangles QRT (labelled as B) and RST (labelled as C). Beng, Liana, and Patrick calculated the area of A, B, and C using the area formulae of rectangle and triangles respectively and then summed them up to get the area of the diagram as 420 cm².

Beng, and Liana, checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle with the dimensions of 15 cm by 8 cm. Beng drew a large rectangle with the dimension of 28 cm by 15 cm and calculated its area by using area formula of rectangle as 420 cm². Liana drew a large rectangle with the dimension of 28 cm by 15 cm. She partitioned the large transformed rectangle into two smaller rectangles with the dimensions of 20 cm by 15 cm and 8 cm by 15 cm and labelled them as A and B respectively. Liana calculated its area by using area formula of rectangle and summed up the area as 420 cm². Patrick checked the answer for area by moving triangle RST under the translation T_{SR} to form a rectangle (labelled as new “B”) with the dimensions of 15 cm by 8 cm. He calculated area of “B” by using area formula of a rectangle, namely $15 \times 8 = 120$ cm². Patrick explained that both methods gave the same answer, namely 420 cm².

When probed to check the answer for the area, Tan used an alternative procedure (alternative method), namely partition-and-sum algorithm to generate an answer which could be used to verify his original answer. Tan used the “cut and paste” transformation to transform Diagram 1 into a “long” rectangle. He calculated the area of Diagram 1 as the area of the “long” rectangle formed using the

area formula of a rectangle where its length and width is 28 cm and 15 cm respectively. Tan got the area of the diagram as 420 cm^2 .

Tan checked the answer for area using the partition-and-sum algorithm to calculate the area of the diagram. He partitioned Diagram 1 into rectangle PQTU and parallelogram QRST. Tan calculated the area of the rectangle using the area formula of a rectangle as 300 cm^2 . He calculated the area of the parallelogram using the area formula of a parallelogram as 120 cm^2 . Tan then summed them up to get the area of the diagram as 420 cm^2 . Tan explained that both methods gave the same answer, namely 420 cm^2 .

Ethical Knowledge

All the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, wrote the measurement unit (without being probed), namely cm, for the answer of the perimeter of Diagram 1 that they have calculated. Likewise, all the PSSMTs, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, also wrote the measurement unit (without probed), namely cm^2 , for the answer of the area of Diagram 1 that they have calculated.

All the PSSMTs have successfully calculated the perimeter of Diagram 1, except Tan. Nevertheless, none of the PSSMTs checked the correctness of the answer for the perimeter. Tan might have spotted his mistake should he checked the answer for the perimeter of Diagram 1. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answer for perimeter.

All the PSSMTs have successfully calculated the area of Diagram 1, except Mazlan and Usha. Nevertheless, none of the PSSMTs checked the correctness of the answer for the area. Mazlan and Usha might have spotted their mistake should they checked the answer for the area of Diagram 1. When probed to check answer, then only all the PSSMTs suggested the strategies that they would use to check the answer for area.

CONCLUSION AND DISCUSSION

Most of the PSSMTs in this study had adequate procedural knowledge of calculating perimeter and area of composite figure. For instance, all but one PSSMT had successfully calculated the perimeter of Diagram 1 in Task 6.1 as 104 cm. Similarly, six out of eight PSSMTs had successfully calculated the area of Diagram 1 in Task 6.1 as 420 cm^2 . Mazlan and Usha confused with the slanted side and the height of the parallelogram in Diagram 1 that they used the slanted side QR as the height ($TR = 8 \text{ cm}$) of the parallelogram. Thus, Mazlan and Usha incorrectly calculated the area of the diagram as 555 cm^2 . This finding concurs with Cavanagh (2008) and van de Walle (2007) who found that students tend to confuse with the slanted side (slanted height) and the height (perpendicular height) of a parallelogram. However, the finding of this study is in contrast with the findings of Baturo and Nason (1996). Baturo and Nason (1996) revealed that preservice primary school teachers in their study had inadequate procedural knowledge of calculating area of the given shapes.

All the PSSMTs in this study understand the general measurement convention that perimeter and area is measured by linear units (such as mm, cm, m, km) and square units (such as mm^2 , cm^2 , m^2 , km^2) respectively. These findings are in contrast with the findings of previous studies (Baturo & Nason, 1996; Cavanagh, 2008; Tierney, Boyd, & Davis, 1990). Tierney, Boyd, and Davis (1990) noticed that many prospective primary school teachers from a teachers college in their study labelled the area measurements in linear units. Likewise, Baturo and Nason (1996) found that several preservice primary school teachers in their study wrote the calculated area measurement in linear unit such as 128 cm. Similarly, Cavanagh (2008) revealed that high school students in his study inappropriately labeled the length of sides in cm^2 or areas in cm on the written test. They did not understand the general measurement convention that length is measured in linear units while area is measured in square units.

None of the PSSMTs in this study check the correctness of the answers for the perimeter and area of Diagram 1. Once getting an answer, they seemed to satisfy that the task was finished. When probed to check answer, then only they suggested the strategies that they would use to check the answers. This finding is in agreement with the finding of previous study (Baturo & Nason, 1996) which found that majority of the preservice primary school teachers in their study had to be prodded towards checking their answers.

The PSSMTs in this study employed two types of strategies to verify their answers for calculating perimeter and area of composite figure in Tasks 6.1, namely recalculating strategy and alternative method. Recalculating strategy refers to strategy using the same method and calculates again while alternative method refers to method that was different from the original method. This finding is slightly contrast with the finding of previous study (Baturo & Nason, 1996) which found that most preservice primary school teachers in their study who attempted to verify their answers did so by recalculating strategy or using the inverse operation. They never think of using an alternative method to verify their answers.

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APPENDIX A

Task 6.1: Rectangle and parallelogram/triangle

(Puts a handout written the following problem in front of the subject). Suppose that one of your Form One students asks you for help with the following problem:

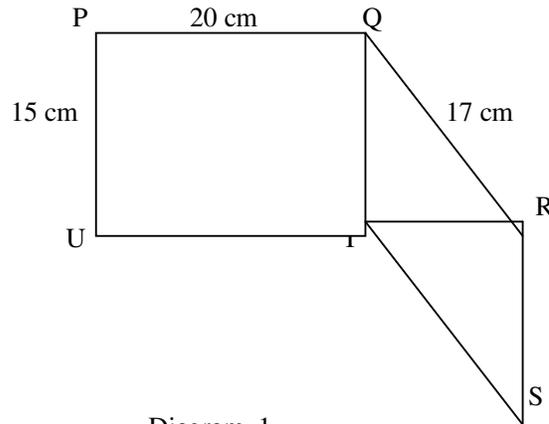


Diagram 1

In Diagram 1, PQTU is a rectangle and QRST is a parallelogram.
UTR is a straight line.

Calculate

- (a) the perimeter of the diagram,
- (b) the area of the diagram.

How would you solve this problem?

Probes:

What do you mean by ____ ?

Could you tell me more about it?

Could you explain your solution?

How did you get that answer?

How would you check your answer?